Physics of the Diffuse Universe

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Overview

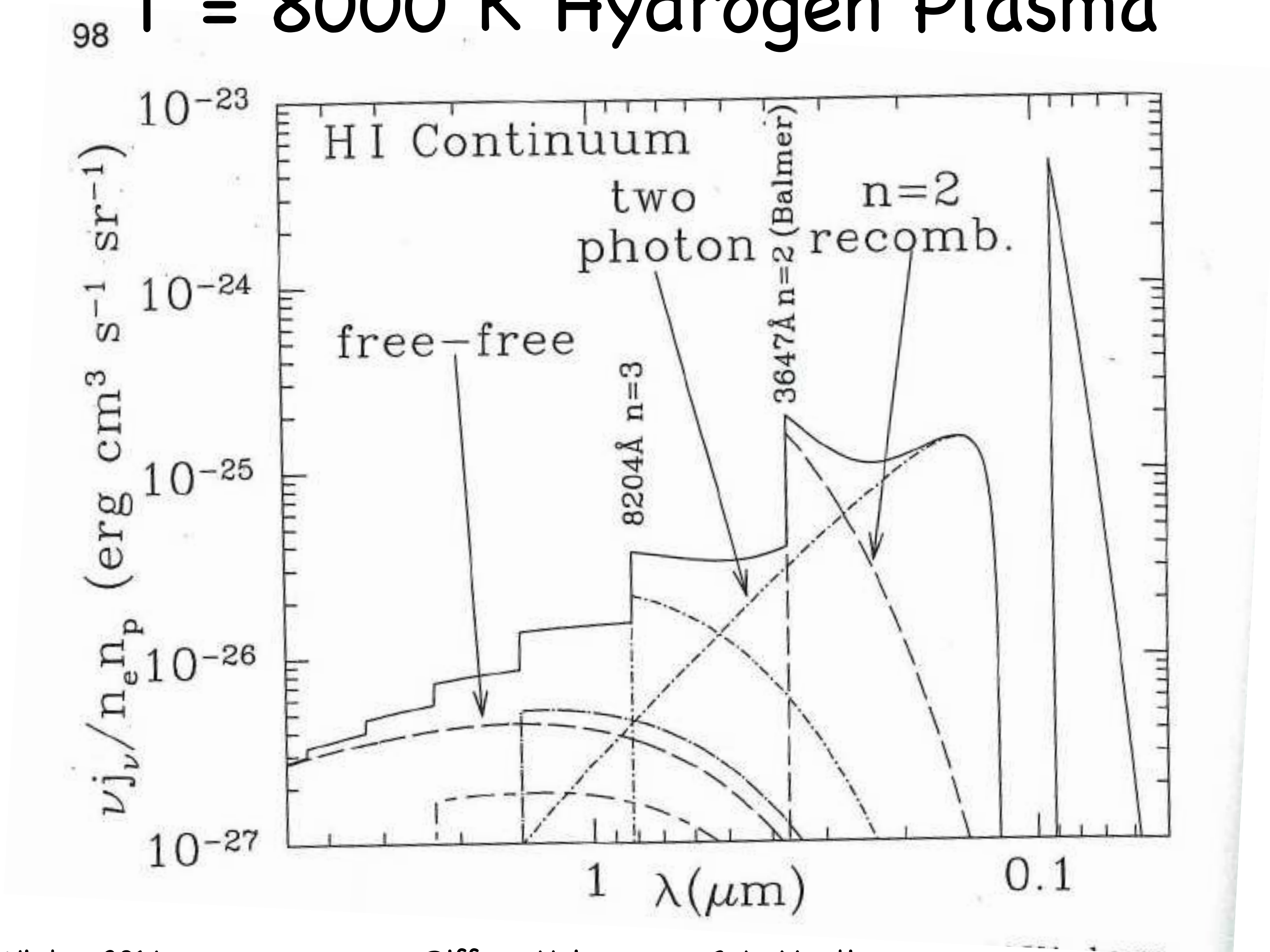
- Microscopic processes control the bulk properties of diffuse gases such as temperature and density.
- These properties determine the evolution of the diffuse gas including the feeding of gas into galaxies and the sites of star formation in the ISM.
- Consider the processes that determine the temperature of diffuse gas.
  - Collisional Ionization Equilibrium
    - Read Draine ch 10, 13, 14.5, 34.1, 34.2 (Figures shown here;

      or see DS ch 5 and 7
  - Photoionization Equilibrium

Outline

- Emission & Absorption by a Thermal Plasma
  - A partially ionized gas where the particle velocity distribution is described by a Maxwellian distribution.
  - Very common in ISM and IGM
  - Temperatures range from 10^3 K to 10^8 K
  - **What does the continuous emission spectrum of a T = 8000 K H plasma look like?**
- Collisional ionization equilibrium
  - Ionization fractions for each element depend only on T
  - Properties of radiative cooling function

Continuous Emission Spectrum: T = 8000 K Hydrogen Plasma



Emission & Absorption by a Thermal Plasma

- Bound – Bound transitions
  - Example of Cooling Rate Calculation
  - Collisional Exictation in C+
- Continuum Processes
  - Free – Free emission (& absorption)
  - Bound – Free emision (& absorption)
- Ionization Processes (& their inverses)
  - Collisional ionization
  - Auger ionization
  - Cosmic ray ionization
  - Photoionization

Cooling by Bound-Bound Transitions

- Example: Radiative decay of collisionally excited e-'s
- A + B --> A + B*
- B* --> B + hv
- Frequent collisions with KE at least as large as the excitation energy
  - H: E ~ 10 eV, which means T > 1e4 K
  - C+: E/k ~ 92 K
  - O: E/k~ 98 K, 228 K, 326 K
  - Si+: E/K~413 K
- Photon emission before the next collision
- Emitted photons are not re-absorbed; I.e., the gas is optically thin in the cooling radiation

Cooling in HI Regions (T~100 K)

- C+ collides with H, e, H2
- H likely the most abundant particle
- 157.68 um
- E/k = 92 K
- ncrit = 49.3 cm-3
- Estimate the cooling rate assuming cooling occurs by excitation of C+ in collisions with H atoms.
- [BB]

Free-Free Emission (& Absorption)

- Power radiated per unit volume scales as $T^{1/2} Z_i^2 n_i n_e$
- Most of the f-f power is near $h\nu \sim kT$
- The radio and microwave f-f spectrum is almost flat, declining with increasing frequency as $\nu^{-0.12l}$
- Attenuation coefficient follows from Kirchhoff's law: $\kappa_\nu = j_\nu / B_\nu(T)$.
  - Strong at low frequencies
  - Negligible in the ISM for $\nu > 10$ GHz
  - HII regions can be optically thick at $\nu < 1$ GHz

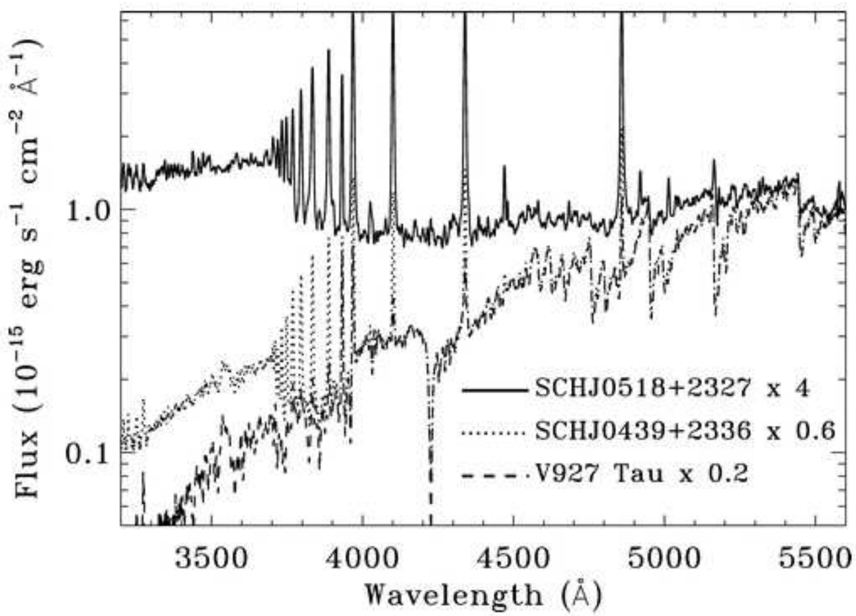
Bound-Free Emission (Radiative Recombination)

- An ion captures an electron.
- Which e- energies are favored?
- This bound state has lower energy than the two free particles, and the excess energy is radiated away.
- A(i+) + e- --> A*([i-1]+) + hv

  A*([I-1]+) --> A([I-1]+) + hv1 + hv2 ...
- The first photon represents the **recombination continuum**
- The radiative cascade in the excited ion produces **recombination lines**

Bound-Free Emission (& Absorption)

- Example: Young star with strong accretion
  - Note the Balmer series (Bound-bound transitions)
  - And the Balmer continuum



- **What is the inverse of radiative recombination called?**

Photoionization (Photoelectric Absorption)

- Photoionization

  A(i) + hν --> A(i+1) + e- + ΔE

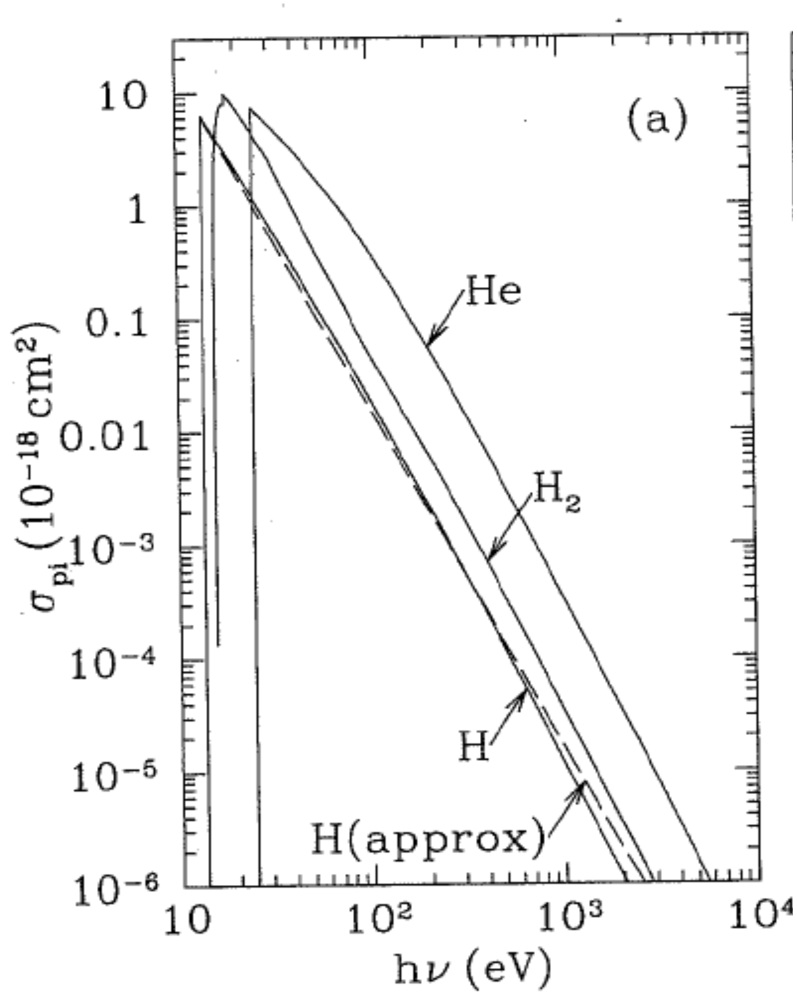
  or

  A(i) + hν --> A*(i+1) + e- + ΔE

  A*(i+1) --> A(i+1) + hν1 + hν2 + …

- Cross Section for photoionization
  - Simple analytic expression for one-electron ions.
    - $\sigma_0 = 6.304 \times 10^{-18}\ Z^{-2}\ \mathrm{cm}^2$, Z is atomic number
    - $\sigma(\nu) \sim \sigma_0\, \nu^{-3}$
  - More complicated with 3 or more e-'s because the resulting ion can be left in a variety of states.
    - Absorption edge at the minimum photon energy for photoionzatin from a shell.
    - Can dominate (despite low abundance) at high energies.

Photoionization Cross Sections: H, H2, He, C, and O



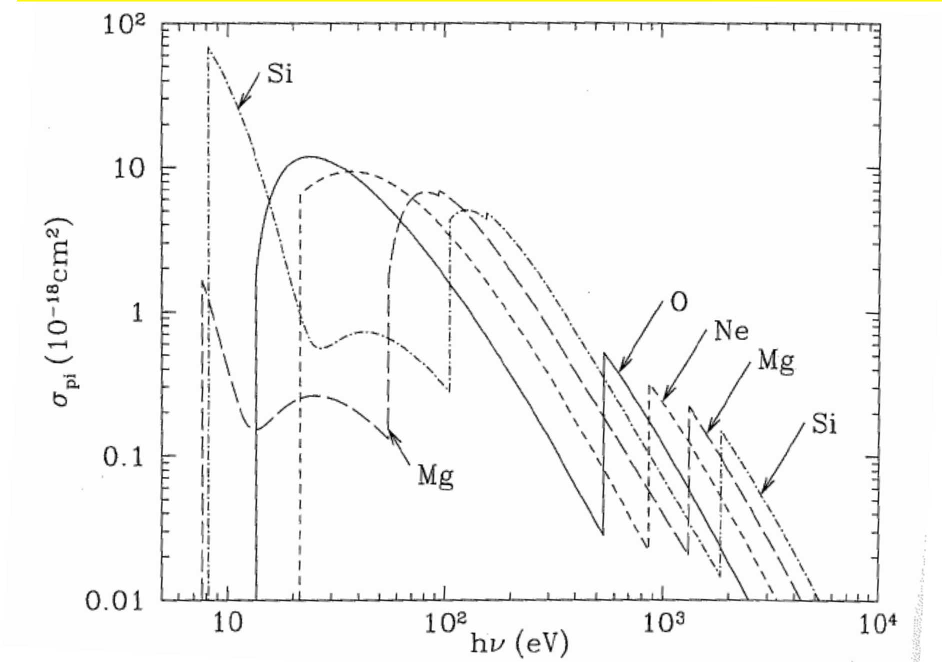
Simple analytic expression for one-electron ions.

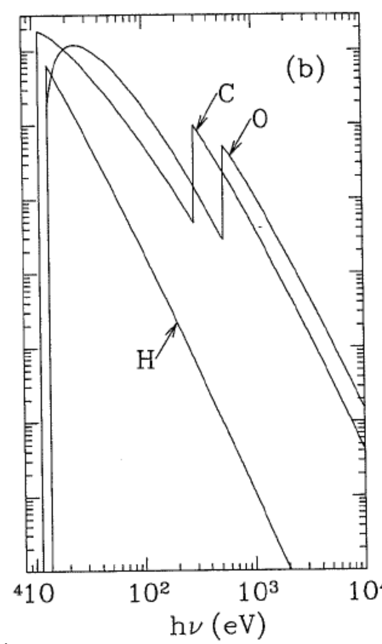
- σ_0 = 6.304 X 10^{-18} Z^{-2} cm^2, Z is atomic number
- $\sigma(\nu) \sim \sigma_0 \nu^{-3}$

Photoionization Cross Sections: O, Ne, Mg, Si

More complicated with 3 or more e-'s because the resulting ion can be left in a variety of states.

- Absorption edge at the minimum photon energy for photoionzatin from a shell.
- Can dominate (despite low abundance) at high energies.





Radiative Recombination Rates

- Described by recombination coefficient
- **Milne Relation** relates the cross section for photoionization (from a given level) to the cross section for recombination (to that same level). Derive it using the **principle of detailed balance**.
- For hydrogen

$$\sigma = 6.3042 \times 10^{-18}\ \mathrm{cm}^2\ (\nu/\nu_0)^{-3.5}$$

$$\alpha = 4.18 \times 10^{-13}\ (T/1e4)^{-0.72}\ \mathrm{cm}^3\mathrm{s}^{-1}$$

Inner Shell Photoionization

- X-ray fluorescence
  - One electron drops down (from L shell) to fill the vacancy (K shell); atom emits a photon (Ka).
- Auger ionization.
  - One electron drops down to fill the hole, and a second electron is promoted to an excited (and unbound) level.
- $X + h\nu \rightarrow (X^{+})^{*} + e^{-} \rightarrow X^{+n} + ne^{-}$ (n .ge. 2)
  - Two high energy e-'s are produced.
  - What are their energies?
  - What is the inverse of Auger ionization?

Dielectronic Recombination

- Often exceeds the radiative recombination rate in high T plasmas
- The e- rarely has enough energy at low T to produce a doubly excited state.
  - Mg II and C III are exceptions in which the dielectronic recombination rate is significant at 10^4 K.
  - Can be important at low T if one of the excited states is a fine-structure state.
    - CII → CI, Si II → Si I, and OIII → OII
- Populates specific energy levels
  - Subsequent radiative decay produces line ratios that differ from those resulting from pure radiative recombination.
  - Example: Dielectronic recombination of C IV produces C III 2296 A line in PNe.

Collisional Ionization

- $X^i + e^- \rightarrow X^{i+1} + 2\,e^- - \Delta E(IP)$
- Cross sections for CI go up when the KE is high enough to leave X^{i+1} in an excited state
- Compare to collisional excitation -- now excitation is to a continuum of levels above the ionization potential instead of to a single level

Collisional Ionization of Heavy Elements

- More than one electron of the target nucleus may be excited
- The unstable atom ejects a 2nd e- (radiationless) then decays to the ground state
- This process is called **autoionization**

Other Ionization Processes

- 3-body
- Charge-exchange
- Cosmic ray ionization
- Dissociative recombination
- Neutralization by grain

Cooling Plasmas

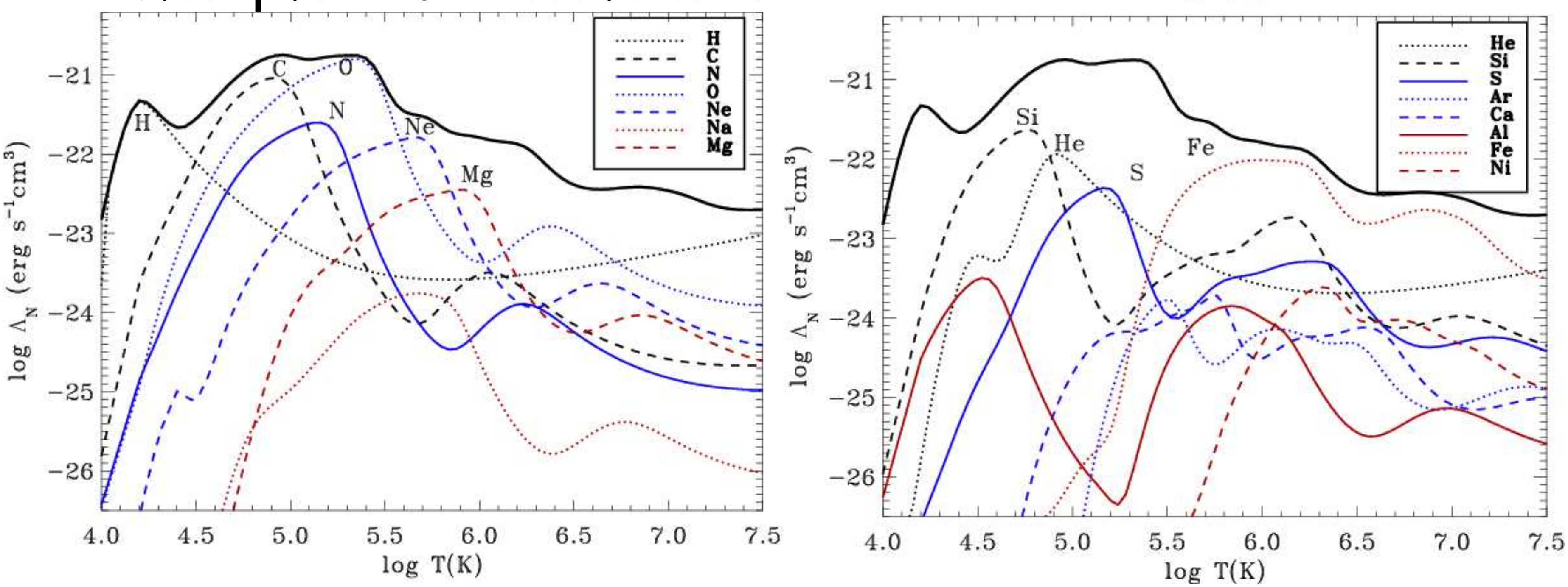
- Collisional Ionization Equilibrium
- Cooling rates
- Cooling timescales
- Non-equilibrium cooling

Collisional Ionization Equilibrium

- Couple the stages of ionization using rates for collisional ionization, recombination, and charge-exchange reactions.
- Coronal approximation simplifies equations.
  - Excitations occur only from the ground state of the ion.
  - Applies at low density.
- Further simplification, ignoring CE reactions gives $n_A(i+1)/ n_A(i) = \alpha_{coll}^{A,i} / \alpha_{rec}^{A,i+1}$
- Dopita & Sutherland 1993 calculate the full ionization balance and plot it against T for most elements. Patterns reflect the shell structure of the atoms.

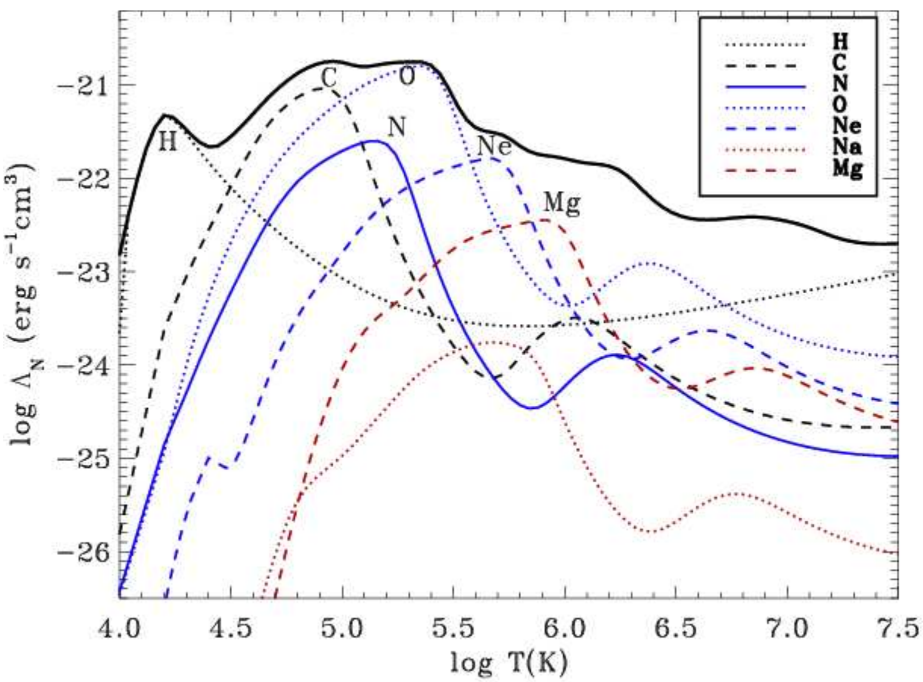
Cooling Plasmas

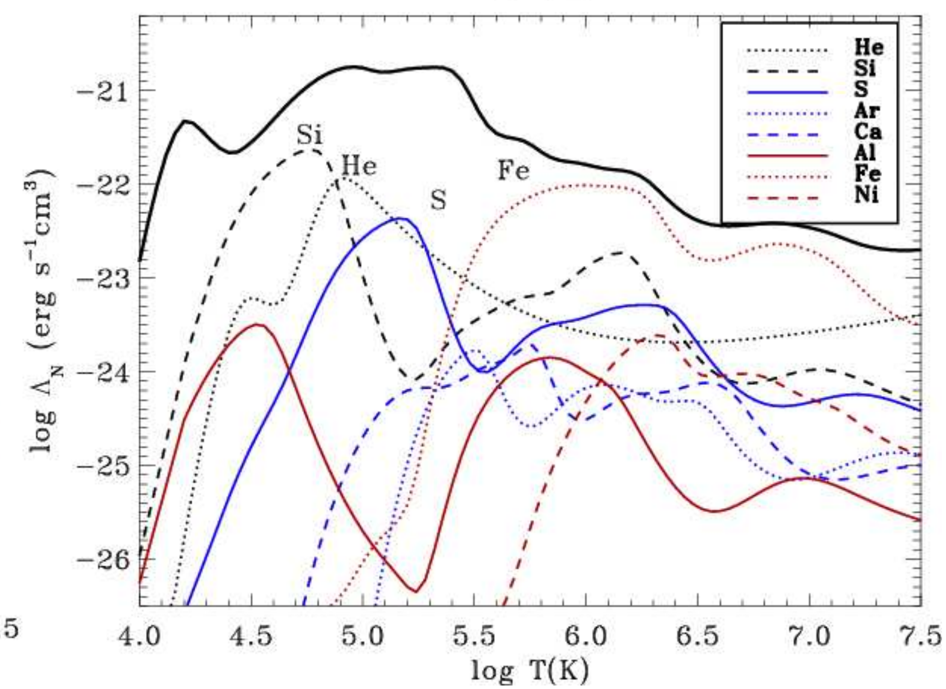
- Cooling Function $Q(n_e,T,Z) = \Lambda n_e n$ describes the total energy lost by a plasma per unit volume per unit time.
- Example: Sutherland & Dopita 1993



Cooling Function Λ

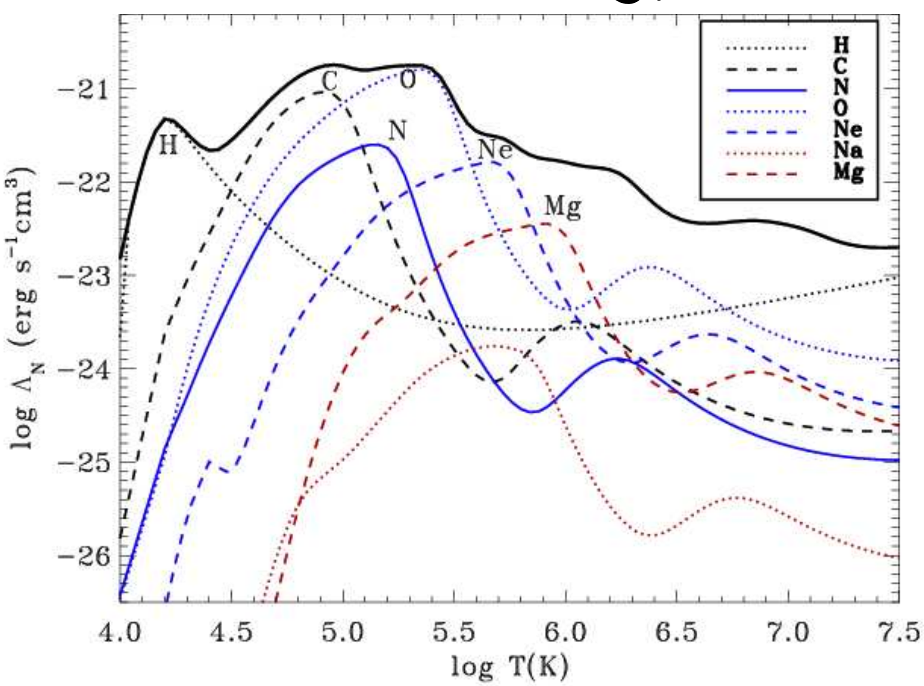
- When H^0 and He^0 present, dominated by collisional excitation for log T ~ 4
- And by He^+ up to log T~ 5.6

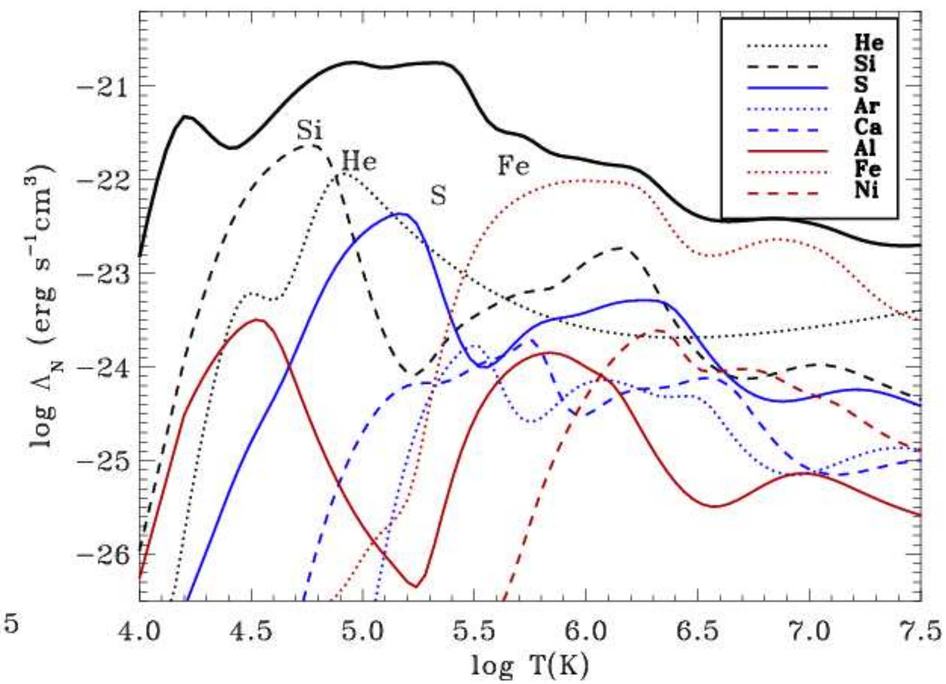




Cooling Function Λ

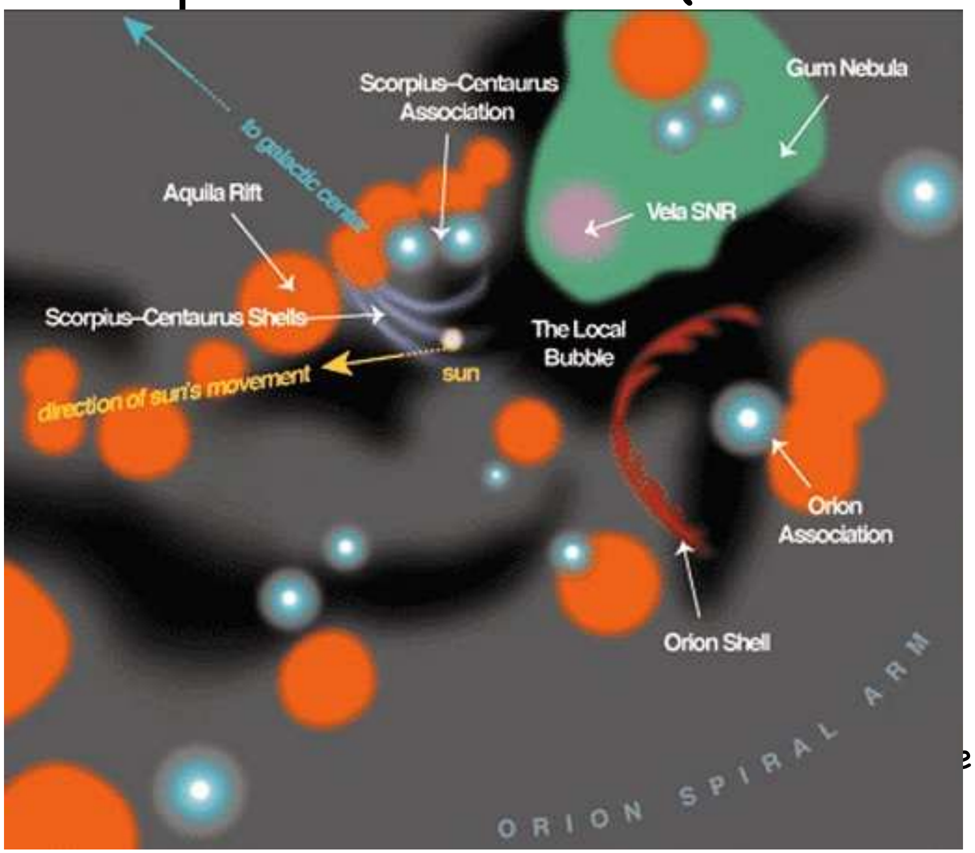
- At log T < 5, forbidden lines of C, N, O, and Ne (in the optical and UV) dominate
- How does Λ change with the metallicity?
- How does energy loss rate change with T?

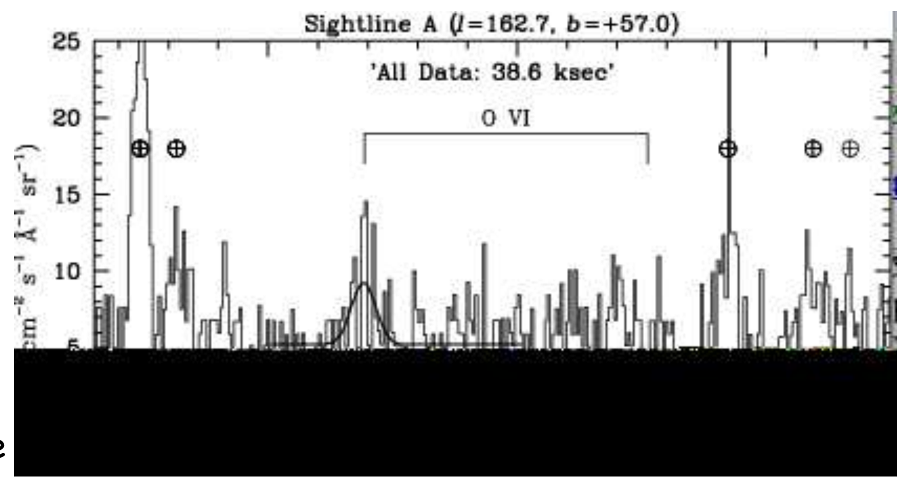




Tracers of "Cooling" Gas

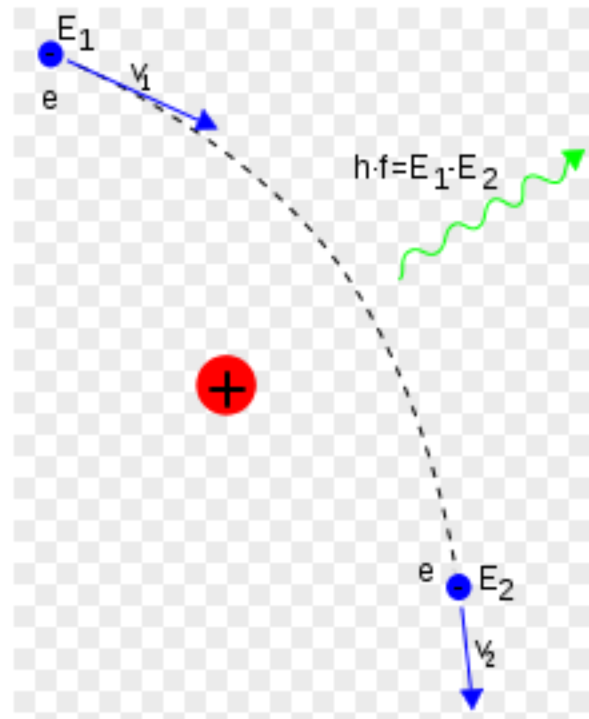
- Between log T = 5 and 6, resonance lines in the EUV dominate dominate Λ
- OVI 1032, 1038 observed with FUSE
- Example: Detection of local interstellar bubble ~ 100 pc from the Sun (Welsh + 2002)





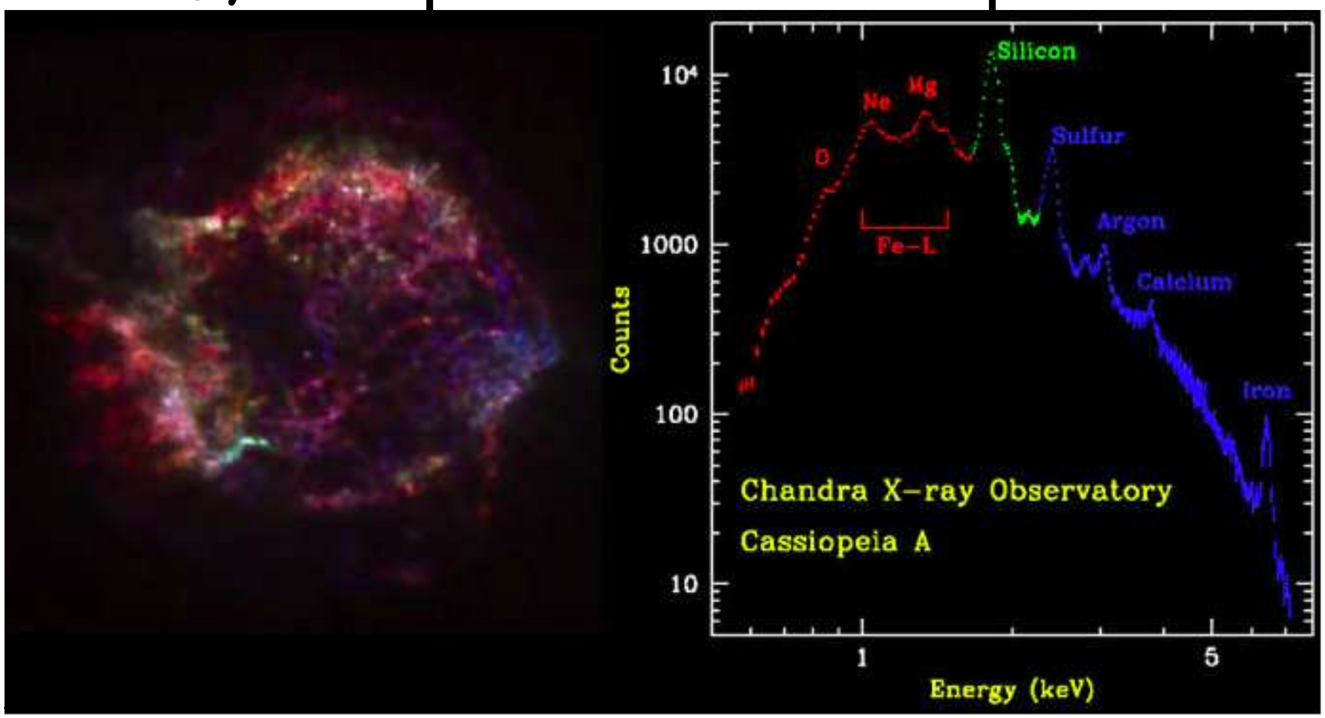
Free-free Cooling in Very Hot Plasmas

- What are examples of hotter plasmas?
- Power radiated by Bremmstrahlung
  - has an exponential cut-off around $h\nu \sim kT_e$
  - See derivation in Rybicki & Lightmann; note dependence on $T^{1/2}$
  - $\Lambda \sim$ (3e-27 ergs cm3) $T^{1/2}$



X-Ray Emitting Plasmas

- Fe becomes the dominant coolant (at X-ray frequencies) for log T > 6
- K shell e's are finally stripped from Fe above log T ~ 8.0; and pure free-free spectrum



Cooling Timescale

- Total heat content of a plasma
  $Q = 3/2\ (n_e + n_{ion})\ kT_e$, assuming $T_e = T_{ion}$
- $t_{cool} \sim Q/\ (\Lambda n^2)$
- Or, fit $\Lambda(T)$ for a more accurate estimate
- For Λ_{ff}, we have
  $t_{cool} \sim 50$ Myr n-1 $T_8^{1/2}$
- Recall the Hubble time is 1.5e10 yr, so gas with n < 3e-3 cm-3 that is heated to 1e8 K will never cool.
- This idea is central to galaxy formation models.

Cooling Time

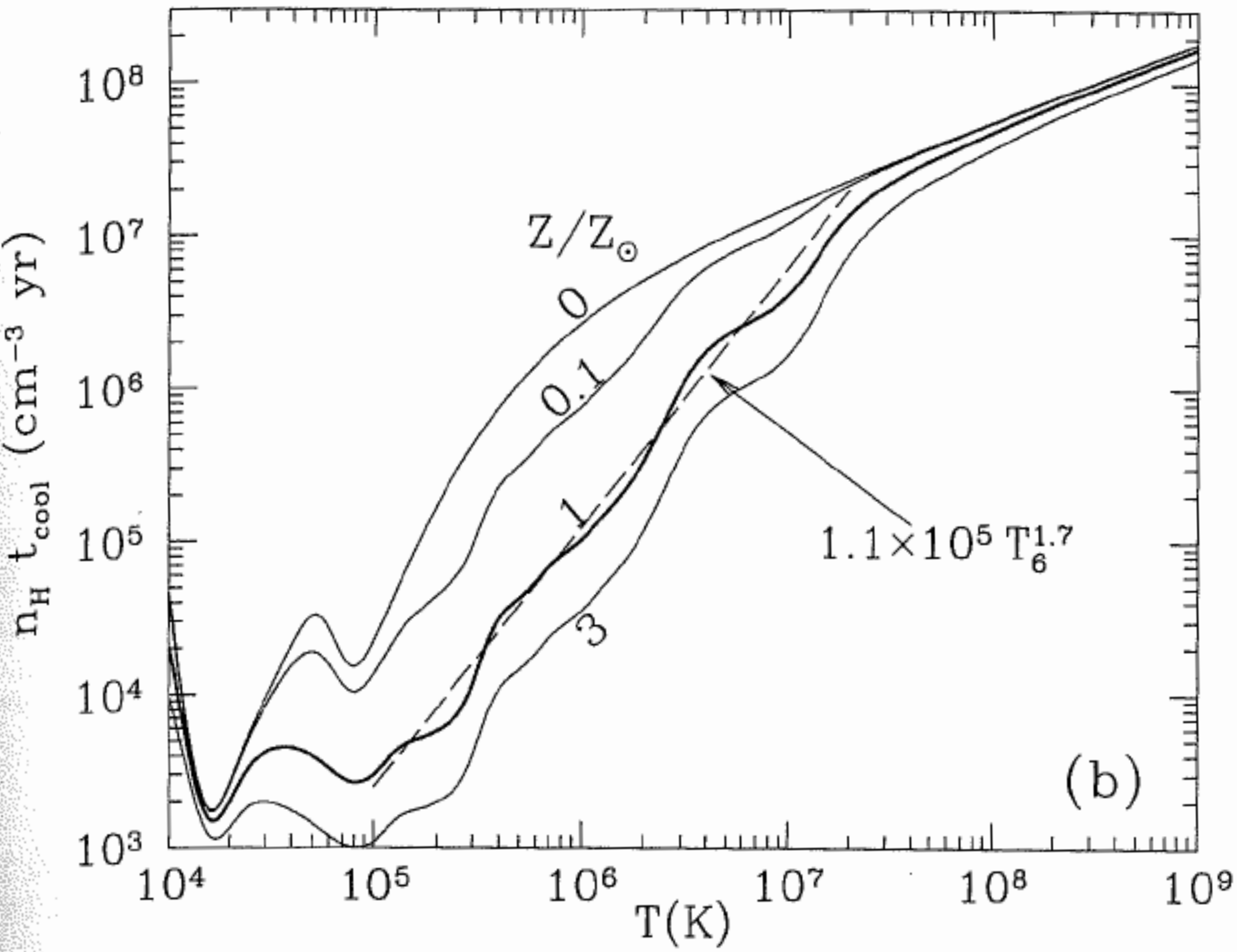


Figure 34.4 $n_{\rm H}t_{\rm cool,isochoric}$ for isochoric cooling with the cooling function from Fig. 34.1, for different metal abundances relative to solar. The dashed line is the approximation (34.4) for $Z \approx Z_\odot$ and 10^5 K $< T < 10^{7.3}$ K.

Nonequilbrium Cooling

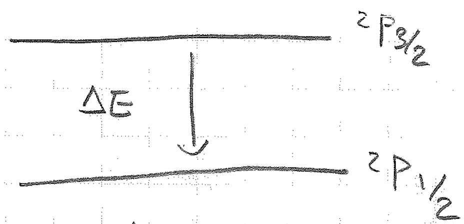
- Sometimes T_{ion} .ne. T_e
- For example, when the plasma cools faster than it can recombine, the gas is left in an over-ionized state relative to the kinetic energy distribution of the particles
- Or, when diffuse gas is suddenly heated, the gas may be under-ionized until collisional excitations 'catch up'.
- Signature - Weak/Strong line cooling from collisionally excited species.

Cooling by Collisional Excitation

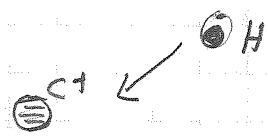
$$A + B \to A + B^*$$

$$B^* \to B + h\nu$$ Photon escapes and removes KE

C^+ [CII] 157.68 μm



$\frac{\Delta E}{k} = 92\,K$



Consider collisions

$$\frac{\text{energy radiated}}{\text{time} \cdot \text{volume}} = \Lambda = n_{c^+,1}\, n_H\, \alpha_{12}\, h\nu$$

$$\alpha_{12} = \alpha_{21} \frac{g_2}{g_1} e^{-\Delta E/kT}$$

[$cm^3 s^{-1}$, where $R_{12} = n_1 n_e \alpha_{12}$]

Found this previously from detailed balance; see DS ch. 3.

$$g_1 = 2J+1 = 2$$

$$g_2 = 4$$

$$\alpha_{21}(H-C^+) = 8 \times 10^{-10}\ cm^3\ s^{-1}$$ From Spitzer Table 4.2

$$\Lambda = n_{c^+,1}\, n_H\ 92K\left(1.38\times10^{-16}\frac{erg}{K}\right)\left(8\times10^{-10}\,cm^3 s^{-1}\right)\frac{4}{2}\, e^{-92/T[K]}$$

$$= n_{c^+,1}\, n_H \left(8.10\times10^{-24}\ erg\ cm^3 s^{-1}\right)$$ @ 100 K

6.43×10^{-24} @ 80K

Assume $n(H^0) \approx n(H)$

$n(C^+_1) \approx n(C)$

Since $n(C) \approx 4\times10^{-4}\, n(H)$

$$\Lambda = n_H^2 \left(3.24\times10^{-27}\ erg\ cm^3 s^{-1}\right)$$

2.5×10^{-27} Agrees w/ Spitzer 6-13

Excitation - Autoionization (Type of collisional ionization)

$$A^{i+} + e^- \;(\text{high } E\ e^-) \rightarrow A^{i+}_* + e^- \;(\text{lower } E\ e^-) - E_1$$

multiple excited e^-s

$$A^{i+}_* \rightarrow A^{(i+1)+}_* + e^- + E_2$$

$$A^{(i+1)+}_* \rightarrow A^{(i+1)+} + h\nu$$

$$A^{i+} + e^- \rightarrow A^{(i+1)+} + 2e^- + h\nu$$

Dielectronic Recombination

$$A^{(i+1)+}(1s, \ldots) + e^- \rightarrow A^{i+}_*(n_1 l_1 ; n_2 l_2)$$

autoionizing state & excited state

$$A^{i+}_*(n_1 l_1 ; n_2 l_2) \rightarrow A^{i+}_*(n_3 l_3 ; n_2 l_2) + h\nu \qquad \text{stabilizes}$$

$$A^{i+}_*(n_3 l_3 ; n_2 l_2) \rightarrow A^{i+}(n_3 l_3 ; n_4 l_4) + h\nu_1 + h\nu_2 + \cdots \qquad \text{radiative cascade}$$

Auger ionization - photoionization from inner K or L shell

$$A^{i+} + h\nu \rightarrow A^{(i+1)+}_{**} + e^- + \Delta E_1$$

$$A^{(i+1)+}_{**} \rightarrow A^{(i+m+1)+}_* + m e^- + \Delta E_2$$

$$A^{(i+m+1)+}_* \rightarrow A^{(i+m+1)+} + h\nu_1 + h\nu_2$$

$$A^{i+} + h\nu \rightarrow e^- + m e^- + A^{(i+m+1)+} + h\nu_1 + h\nu_2 + \cdots$$

Draine ch.27
Osterbrock, ch. 3
DS ch. 9

Temperature of a steady-state nebula is fixed by equilibrium between heating rate and cooling rate.

U = Energy density of e^- $= \frac{3}{2} n_e k T_e$ ergs/cm^3

G = Energy gained by e^-s per unit time per unit volume

L = Energy lost by e^-s per unit time per unit volume

$$\frac{dU}{dt} = G - L$$
$$= 0$$

Heating Processes

- Photoionization $\quad A + h\nu \rightarrow A^+ + e^-(v)$
- Collisional De-excitation $\quad A^* + e^-(v_1) \rightarrow A + e^-(v_2)$, $v_2 > v_1$
  (If density sufficiently high.)

Cooling Processes

- Recombination $\quad A^+ + e^- \rightarrow A + h\nu$ $\quad$ Photon carries energy away!
- Collisional Excitation $\quad A + e^-(v_1) \rightarrow A^* + e^-(v_2)$, $v_2 < v_1$
- Free-free Radiation (bremsstrahlung) $\quad A^+ + e^-(v_1) \rightarrow A^+ + e^-(v_2)$, $v_2 < v_1$
  e^- acceleration by ions
  $$L_{FF}(Z) = 1.42 \times 10^{-27} Z^2 T^{1/2} g_{ff} n_e n_{H^+} \text{ ergs/cm}^3\text{/s}$$
  small compared to L_{coll}; important in pure H nebula

Note – in __Ionization Equilibrium__

$$\boxed{\text{Photoionization Rate}} = \boxed{\text{Recombination Rate}}$$

$\frac{1}{2} m \langle v_P \rangle^2$ mean energy of newly created photo-electron

$\frac{1}{2} m \langle v_R \rangle^2$ mean energy that disappears per recombination

[Net Gain in Energy of e^- gas per ionization] = Mean energy new photoelectron − Mean energy carried away per recombination = Rate Energy Lost by Radiation

In thermal equilibrium

- Energy Input by Photoionization

Consider pure H nebula

<u>Heating rate</u>

per absorbed photon is $\frac{1}{2}mv^2 = h\nu - h\nu_0$, and the average over absorbed photons is

$$G = \int_{\nu_{LL}}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu (h\nu - h\nu_0)\, n_H \, d\nu \qquad \text{ergs/s/cm}^3$$

* Note - Rate of photoelectron creation depends on strength of radiation field

- But the mean energy of a newly created photoelectron depends only on the shape of ionizing source spectrum

$$\langle \tfrac{1}{2} m_e v^2 \rangle = \langle \tfrac{3}{2} k T_* \rangle = \frac{\text{Heating Rate}}{\text{Ionization Rate}}$$

$$= \frac{\int_{\nu_{LL}}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu (h\nu - h\nu_0)\, d\nu}{\int_{\nu_{LL}}^{\infty} \frac{4\pi J_\nu}{h\nu} \sigma_\nu \, d\nu}$$

HW Problem → Osterbrock "It is easy to show that $T_i \sim T_*$ so long as $kT_* < h\nu_0$."

Example: Blackbody Source

$$4\pi J_\nu = \pi B_\nu \propto \frac{\nu^3}{e^{h\nu/kT} - 1} \propto \nu^3 e^{-h\nu/kT}$$

$$h\nu_0 = 13.6 \text{ eV}$$

$$kT_* \sim \frac{(1.38 \times 10^{-16} \text{ erg/K})(30{,}000 \text{ K})}{1.6 \times 10^{-12} \text{ erg/eV}} \sim 2.6 \text{ eV}$$

Use $\sigma_\nu = \sigma_0 \left(\frac{\nu_0}{\nu}\right)^3$ cross section

$$\langle Ee \rangle = \frac{\int_{\nu_{LL}}^{\infty} \frac{h\nu}{h\nu}\frac{1}{\nu^3}\nu^3 e^{-h\nu/kT}\,d\nu - \int_{\nu_{LL}}^{\infty}\frac{h\nu_0}{h\nu}\frac{1}{\nu^3}\nu^3 e^{-h\nu/kT}\,d\nu}{\int_{\nu_{LL}}^{\infty}\frac{1}{h\nu}\frac{1}{\nu^3}\nu^3 e^{-h\nu/kT}\,d\nu}$$

$$= \frac{\int_{\nu_{LL}}^{\infty} e^{-h\nu/kT}\,d\nu - \int_{\nu_{LL}}^{\infty}\frac{\nu_0}{\nu} e^{-h\nu/kT}\,d\nu}{\int_{\nu_{LL}}^{\infty}\frac{1}{h\nu} e^{-h\nu/kT}\,d\nu}$$

REM
· Cancel coeffs for $d\nu$.
· keep ν_0 !

Let $x = \frac{h\nu}{kT}$ $\qquad dx = \frac{h}{kT}\,d\nu$

$$= \frac{\int_{\nu_{LL}}^{\infty} e^{-x}\,dx - \frac{h\nu_0}{kT}\int_{\nu_{LL}}^{\infty}\frac{1}{x}e^{-x}\,dx}{\frac{1}{h}\frac{h}{kT}\int_{\nu_{LL}}^{\infty}\frac{1}{x}e^{-x}\,dx}$$

$$= \frac{kT\int_{\nu_{LL}}^{\infty} e^{-x}\,dx - h\nu_0\int_{\nu_{LL}}^{\infty}\frac{1}{x}e^{-x}\,dx}{\int_{\nu_{LL}}^{\infty}\frac{1}{x}e^{-x}\,dx}$$

$$= \frac{kT\left[-e^{-x}\right]_{\nu_{LL}}^{\infty}}{\int_{\nu_{LL}}^{\infty}\frac{1}{x}e^{-x}\,dx} - h\nu_0$$

REM – Just do the nasty integral once!

Use $\int_{x_1}^{\infty}\frac{e^{-x}}{x}\,dx = \frac{e^{-x_1}}{x_1}\left(1 - \frac{1}{x_1} + \frac{2}{x_1^2} + \cdots\right)$

$$= \frac{kT\,\cancel{e^{-x_{LL}}}}{\frac{\cancel{e^{-x_{LL}}}}{x_{LL}}\left(1 - \frac{1}{x_{LL}} + \frac{2}{x_{LL}^2} + \cdots\right)} - h\nu_1$$

REM $x = \frac{h\nu}{kT}$ is large!
$\therefore \frac{1}{x}$ is small.

(cont'd.)

$$\langle E_e \rangle = kT\,\frac{h\nu_{LL}}{kT}\left(1-\frac{1}{x_1}+\frac{2}{x_1^2}\right)^{-1} - h\nu_0$$

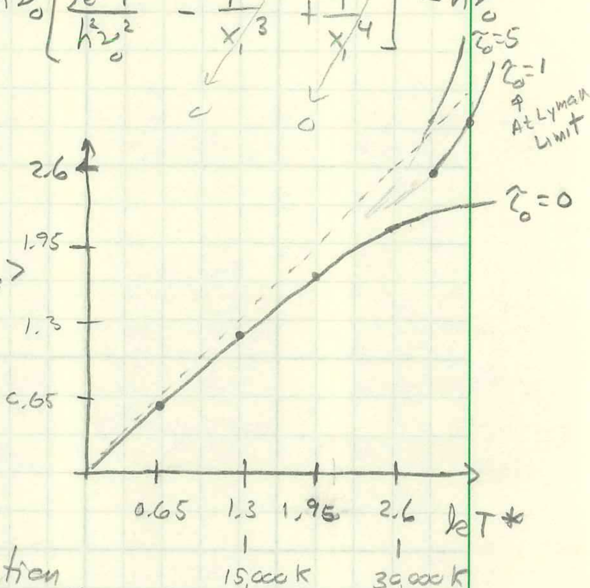
use $(1+z)^{-1} = 1 - z + z^2$, $z \ll 1$

$$= h\nu_0\left(1+\left[\frac{1}{x_1}-\frac{2}{x_1^2}\right]+\left[\frac{1}{x_1}-\frac{2}{x_1^2}\right]^2\right) - h\nu_0$$

$$= h\nu_0\left[1+\frac{kT}{h\nu_0}-\frac{2k^2T^2}{h^2\nu_0^2}\right] + h\nu_0\left[\frac{k^2T^2}{h^2\nu_0^2} - \cancel{\frac{4}{x_1^3}}_{\,0} + \cancel{\frac{4}{x_1^4}}_{\,0}\right] - h\nu_0$$

$$= kT - \frac{2k^2T^2}{h\nu_0} + \frac{k^2T^2}{h\nu_0}$$

$$\therefore \quad \boxed{\left\langle \tfrac{3}{2}kT_i \right\rangle = kT^*\left[1-\frac{kT^*}{h\nu_0}\right]}$$



Note - For hotter stars, larger fraction of photons with small photoionization cross section. $\langle E_e \rangle$ still increases with kT^* but not linearly.

$T_e \sim T_*$ pure H nebula

$T_e < T_*$ coll. ioniz. metals

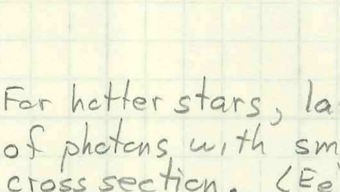
Note - T_i Doesn't depend on geometrical dilution of radiation field.

Does depend on how shape of spectrum is modified by absorption.

We considered the emergent blackbody atmosphere.

What really happens at larger distances from the star?

Prelim question →



Radiation nearest the ionization threshold is most severely attenuated.

⇒ harder spectrum ⇒ higher mean energy per photo-electron further from star.

• Energy Loss by Recombination

+ Since recombination cross sections $\propto v^{-2}$, the e^-s of lower K.E. are preferentially captured.

Mean energy of captured e^- is

$$\langle E_R \rangle < \tfrac{3}{2} kT .$$

← HW/Exam Problem: Show that mean K.E. of a recombining e^- is kT.

+ Kinetic Energy lost by e^- gas per unit time per unit volume is

$$L_R \sim \text{Recombination Rate} \times \text{Mean energy recombining } e^-$$

$$L_R \propto n_e n_{H^+} \alpha_B \langle v^2 \rangle$$

$$\propto n_e n_{H^+} \frac{1}{\sqrt{T_e}} T_e \propto \sqrt{T_e}$$

Thermostat!

NB: As temperature increases cooling rate increases. The faster e^-s have a more difficult time recombining but they take a larger chunk of K.E. with them.

+ In practice use b-f cooling rate given by

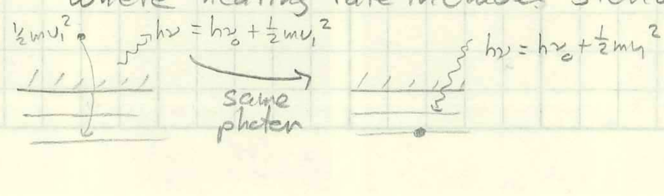
$$L_R(H) = L_{O.T.S.} = n_e n_p kT \beta_B(H, T)$$

kinetic-energy weighted recombination coefficient, O'brock Table 3.2

+ Why $\beta_B = \sum_{n=2}^{\infty} \sum_{L=0}^{n-1} \beta_{nL}(H^0, T)$ with $n=2$?

$$G_{O.T.S.} = L_{O.T.S.}$$

where heating rate includes stellar radiation field only.

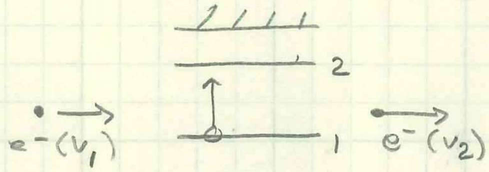


- Energy loss by Collisionally Excited Line Radiation

  + Low-lying energy levels of O^+, O^{++}, N^+ have excitation energies $\sim kT_*$.

    H most abundant but $\chi_{12} = 10\,eV$.
    $kT \sim 1\,eV$ at $T = 10{,}000\,K$.

  + Example: Excitation of ion to level 2.



$v_2 < v_1$ i.e. cooling

Cross section for excitation is zero below threshold $h\nu_{12}$.

$\sigma_{12} \propto v^{-2}$ not too far below threshold because of focusing of Coulomb force.

(Note: Cross sections for neutral atoms do not scale as v^{-2}: Osterbrock p. 65)

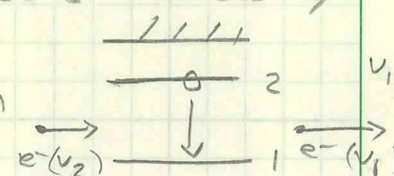
Convenient form to express collision cross section

$$\sigma_{12}(v) = \frac{\pi \hbar^2}{m^2 v_1^2} \frac{\Omega(1,2)}{\omega_1} \quad \text{for } \tfrac{1}{2} m v^2 > \chi_{12}$$

statistical weight of lower level.

  + Principle of Detailed Balance says in Thermodynamic Equilibrium each microscopic process is balanced by its inverse.

    Collisional De-Excitation Cross Section



$v_1 > v_2$

$$\tfrac{1}{2} m v_1^2 = \tfrac{1}{2} m v_2^2 + \chi_{12}$$

Exercise to derive

Rate Collisional Excitations (e⁻ v_1 to $v_1 + dv_1$) = Rate Collisional De-excitations that produce e⁻s in same vel. range

$$\sigma_{12} v_1 n_1 n_e f(v_1) dv_1 = \sigma_{21} v_2 n_2 n_e f(v_2) dv_2$$

In Thermodynamic Equilibrium, Boltzmann eqn gives relative level populations –

$$\frac{n_2}{n_1} = \frac{\omega_2}{\omega_1} e^{-\chi_{12}/kT}$$

HW? ⟹ Derive $\omega_1 v_1^2 \sigma_{12}(v_1) = \omega_2 v_2^2 \sigma_{21}(v_2)$

Express de-excitation cross section in terms of collision strength $\Omega(1,2)$

$$\sigma_{21} = \frac{\pi \hbar^2}{m^2 v_2^2} \frac{\Omega(1,2)}{\omega_2}$$

+ Cooling rate LDL

Every collisional excitation is followed by the emission of a photon and

$$L_c = n_e n_1 h\nu_{12} q_{12} \quad \text{where} \quad [\text{erg/cm}^3/\text{s}]$$

$$q_{21} = \int_0^\infty v \sigma_{21}(v) f(v) dv \qquad \text{cm}^3 \text{s}^{-1}$$

$$= \frac{8.629 \times 10^{-6}}{\sqrt{T}} \frac{\Omega(1,2)}{\omega_2} \qquad [\text{cm}^3 \text{s}^{-1}]$$

v dv

$10^{-15} \cdot 10^{5} \frac{\text{cm}}{\text{s}} \cdot 10^{5} \frac{\text{cm}}{\text{s}}$

Note $q_{21} \sim v \cdot \sigma \cdot v \sim \sigma v^2 \sim (10^{-15} \text{cm}^{-2}) \cdot (10^{5} \frac{\text{cm}}{\text{s}})^2$

$$\sigma \sim 10^{-15} \frac{\Omega}{\omega} \text{ cm}^2$$

Rate coefficient for collisional excitation

$$q_{12} = \frac{\omega_2}{\omega_1} q_{21} e^{-\chi/kT}$$

+ Cooling Rate HDL

Cooling rate is reduced by collisional de-excitation.

Rate level 2 is populated = Rate level 2 is de-populated — steady state

$$q_{12} n_1 n_e = n_2 A_{21} + q_{21} n_2 n_e$$

$$\frac{n_2}{n_1} = \frac{q_{12} n_e}{A_{21} + q_{21} n_e}$$

$$= \frac{1}{A_{21}} \left[\frac{q_{12} n_e}{1 + \frac{q_{21} n_e}{A_{21}}} \right]$$

$\frac{q_{12}}{A_{21}} n_e$

$$= \frac{q_{12} n_e}{A_{21}} \left[\frac{1}{1 + \underbrace{\frac{q_{21} n_e}{A_{21}}}} \right]$$

$\frac{\text{collisional de-excite rate}}{\text{radiative de-excit rate}}$

$$L_c = n_2 h\nu_{21} A_{21}$$

$$= n_1 n_e q_{12} h\nu_{12} \left[\frac{1}{1 + \frac{q_{21} n_e}{A_{21}}} \right]$$

$\xrightarrow{\text{low } n_e}$ $n_e n_1 q_{12} h\nu_{21}$

$\xrightarrow{\text{high } n_e}$ $\underbrace{n_1 \frac{\omega_2}{\omega_1} e^{-\chi/kT}}_{n_2} A_{21} h\nu_{21}$ The thermodynamic cooling rate

For n_e large

REM $q_{12} = \frac{\omega_2}{\omega_1} q_{21} e^{-\chi/kT}$

$\frac{A_{21}}{n_e q_{21}}$

$n_1 n_e \frac{\omega_2}{\omega_1} q_{21} e^{-\chi/kT}$

Point: Collisional de-excitation reduces the cooling rate.

Note: Critical Density $\ni$ $q_{21} n_{crit} = A_{21}$

Table 3.11 Osterbrock – Critical Densities $n_{crit} = \frac{\sum'_{j<i} A_{ij}}{\sum'_{j\neq i} q_{ij}}$

OII	$^2D_{3/2}$	1.6×10^4 cm^{-3}
OII	$^2D_{5/2}$	3.1×10^3 cm^{-3}
OIII	$^1D_2$	7.0×10^5 cm^{-3}
OIII	$^3P_2$	3.8×10^3
OIII	$^3P_1$	1.7×10^3

- Resulting Thermal Equilibrium

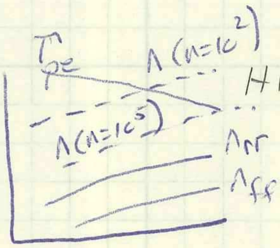
$$\underbrace{G - L_R}_{\text{effective heating rate from photoionization.}} = L_{FF} + L_c$$

The collisionally excited radiative cooling term is sum over terms like LDL, HDL for all transitions.

LDL - All G and L terms $\propto n_e$

So T_e independent of total density.

Depends on ionic abundances

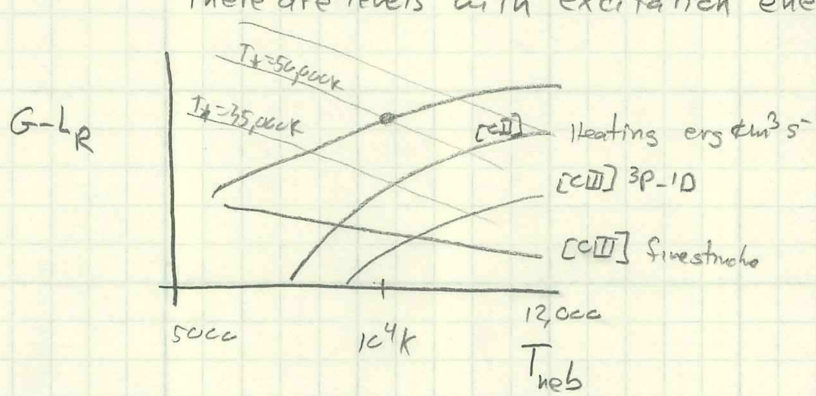


HDL - Cooling rate decreased when collisional de-excitation becomes important, so equilibrium temperature somewhat increased.

Osterbrock Fig. 3.2 - LDL

① Heating Rate depends on T^*

② Total L_c rises with T as long as there are levels with excitation energy $\chi > kT$



③ $G - L_R$ is decreasing with T_{neb} because $L_R(H) \propto kT \cdot \alpha \propto \sqrt{T}$

Osterbrock Fig. 3.3 → HDL → higher equilibrium temperature