Calculated Spectrum: Additional Components

- Recombination lines
- Nebular Continuum
- Radio spectrum

Recombination Lines

- These bright emission lines can be detected from the most distant galaxies and AGN. Their spectrum reveals the physical conditions (T,n) of the gas in those environments.
- HI Lines
- HeI and HeII Lines

HI recombination lines

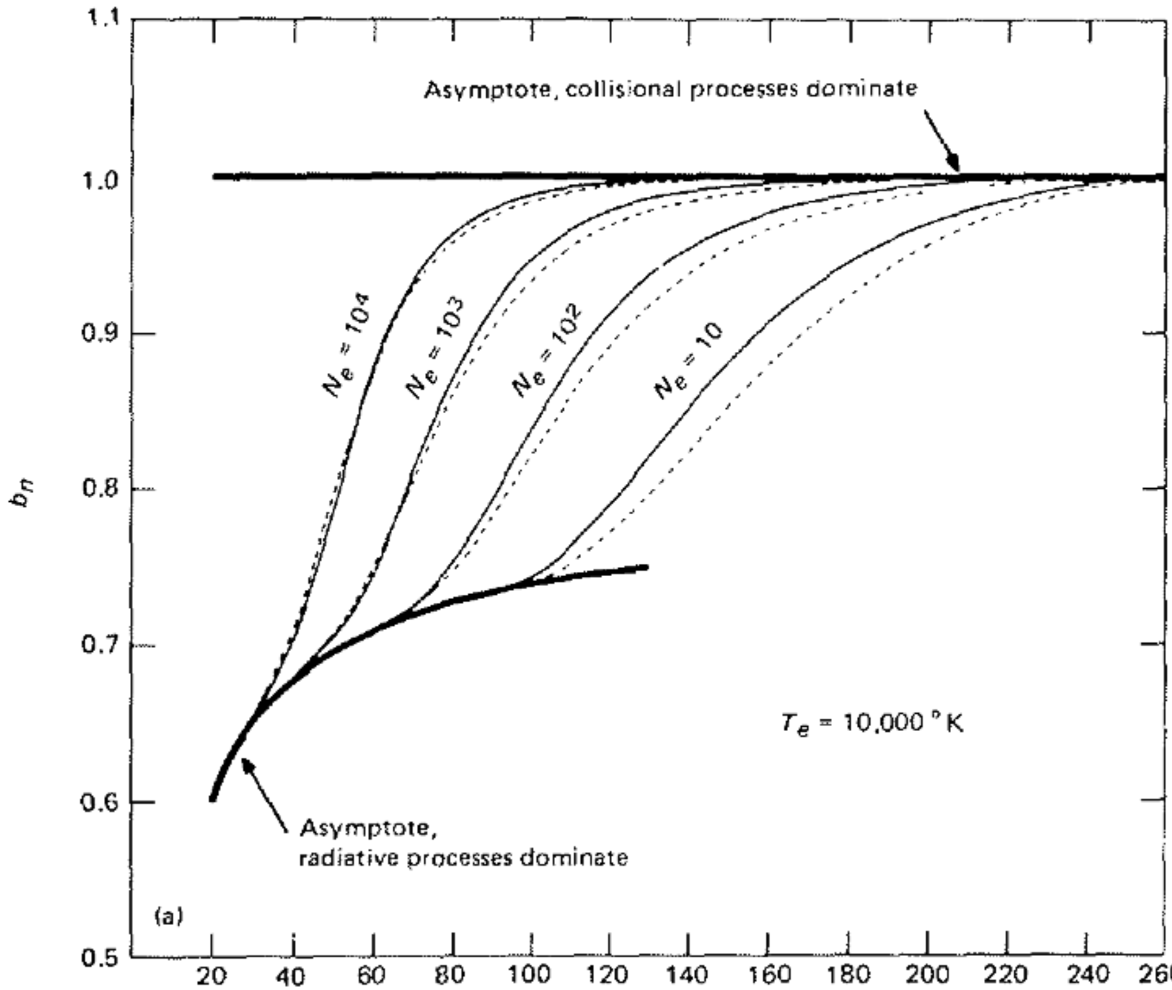
- Case A recombination is rare
  - τ(Lya) ~ 10,000 τ(912)
  - τ(Lyb) ~ 1,000 τ(912)
- Case B recombination
  - Photons emitted in the Lyman series are quickly re-absorbed by H(1s)
- Statistical equilibrium [BB]

HI Recombination Lines (cont'd)

- Balmer decrement [BB]
  - Case B values
  - Measuring interstellar extinction
- What are some physical causes of departures from Case B values?
- Transition from Case A to Case B, see Osterbrock S 4.5

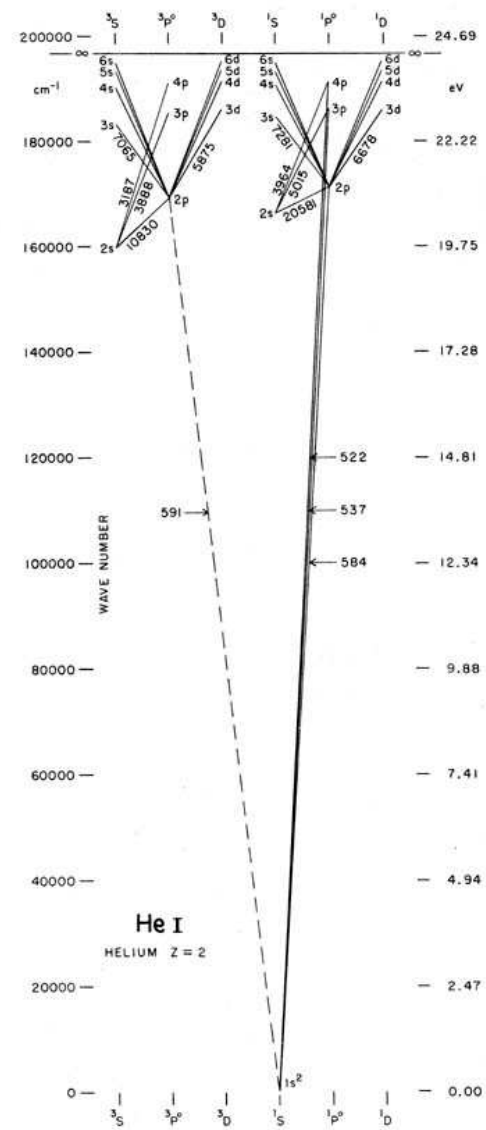
Departure Coefficients

- b as a function of principle quantum number
- *Sejnowski, T. J., & Hjellming, R. M. 1969, ApJ, 156, 915*



Recombination lines of He

- *What about He+?*



Nebular Continuum

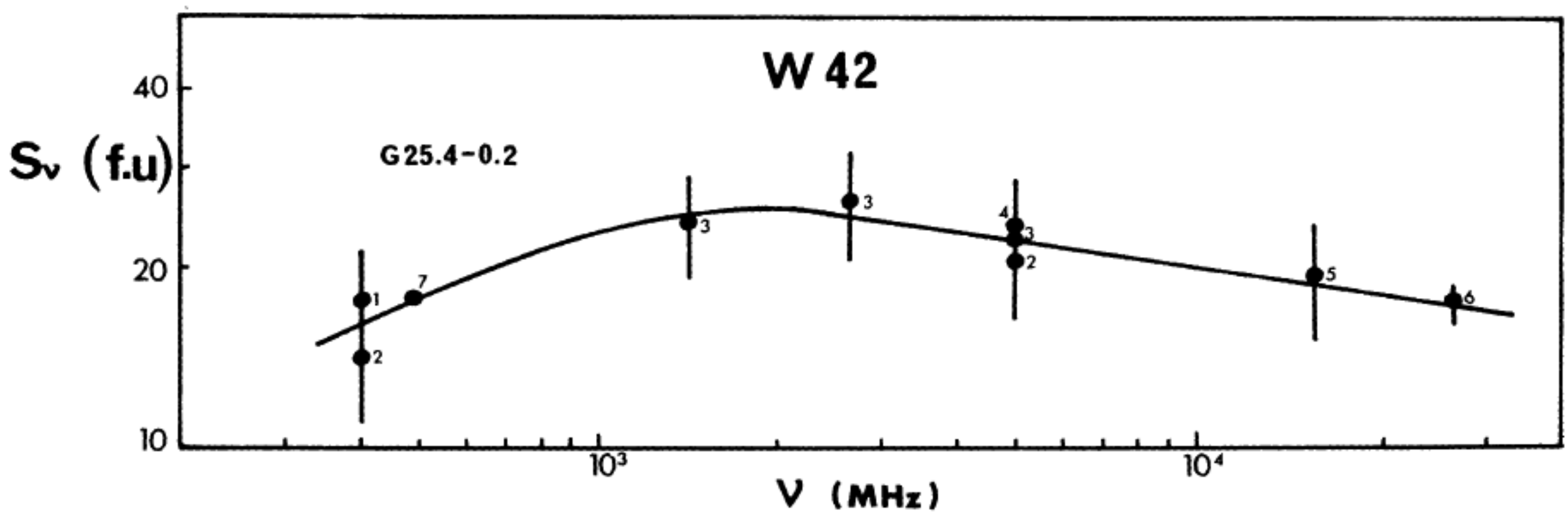
- Free-bound emission from recombination
- Free-free emission from e- scattering off the protons
- 2 photon HI continuum

Radio Emission from HII Regions

- Continuum due to free-free emission
  - $h\nu << kT$
  - Free-free Gaunt factor
    - g_{ff} ~ 1- 1.5 in UV/opt
    - g_{ff} ~ 10 at ν = 1 GHz and T = 1e4 K
  - Free-free radiation is also absorbed at low radio frequencies
    - Example: ne ~ np ~ 100 cm-3, T = 1e4 K, and l = 10 pc ==> tau(ff) ~ 1 at 200 MHz
    - PNe with ne ~ 3000 cm-3; l = 0.1 pc ==> tau(ff) = 1 at 600 MHz
- The spectrum rises as ν^2 at low frequencies (optically thick regime), flattens (tau ~ 1), and falls like ν^{-1} at high frequency [BB]

Turn-Over Frequency

- Observation at low frequency where nebular is optically thick ==> T_B measures T_e directly
- Turn-over frequency gives n_e
- *Below: Goudis 1977 Ap&SS 47, 109*

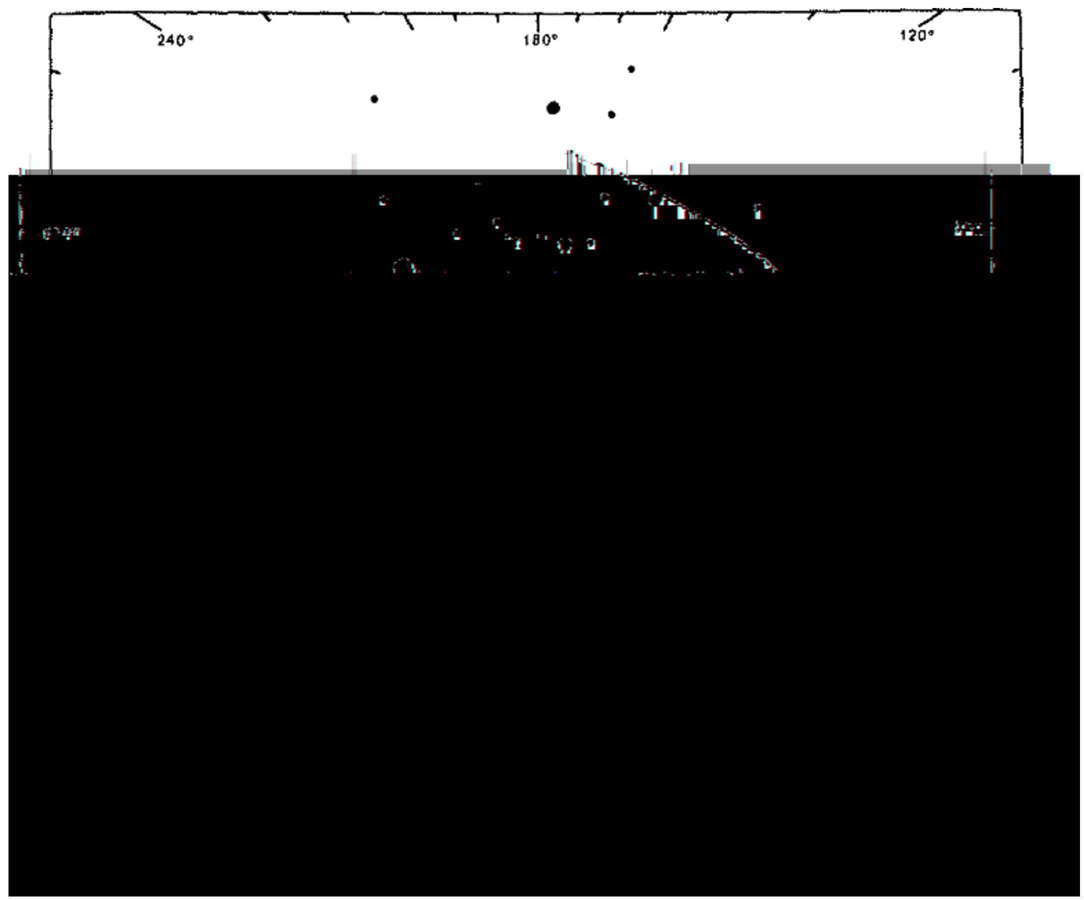


Radio Recombination Lines

- What is the frequency of H106α? H106β?
- Maximum principle quantum number for H is ~ 740 for typical nebular conditions (1000 cm-3 and 1e4 K0. Why?
- The level populations in these very high n states are usually very small, ~10^{-5} of the ground state.
- Observed in Galactic HII regions; quite faint for extragalactic work
- In LTE, the ratio of line to continuum strength can provide estimates of the electron temperature in nebulae.
- **Name an important advantage over optical recombination lines.**

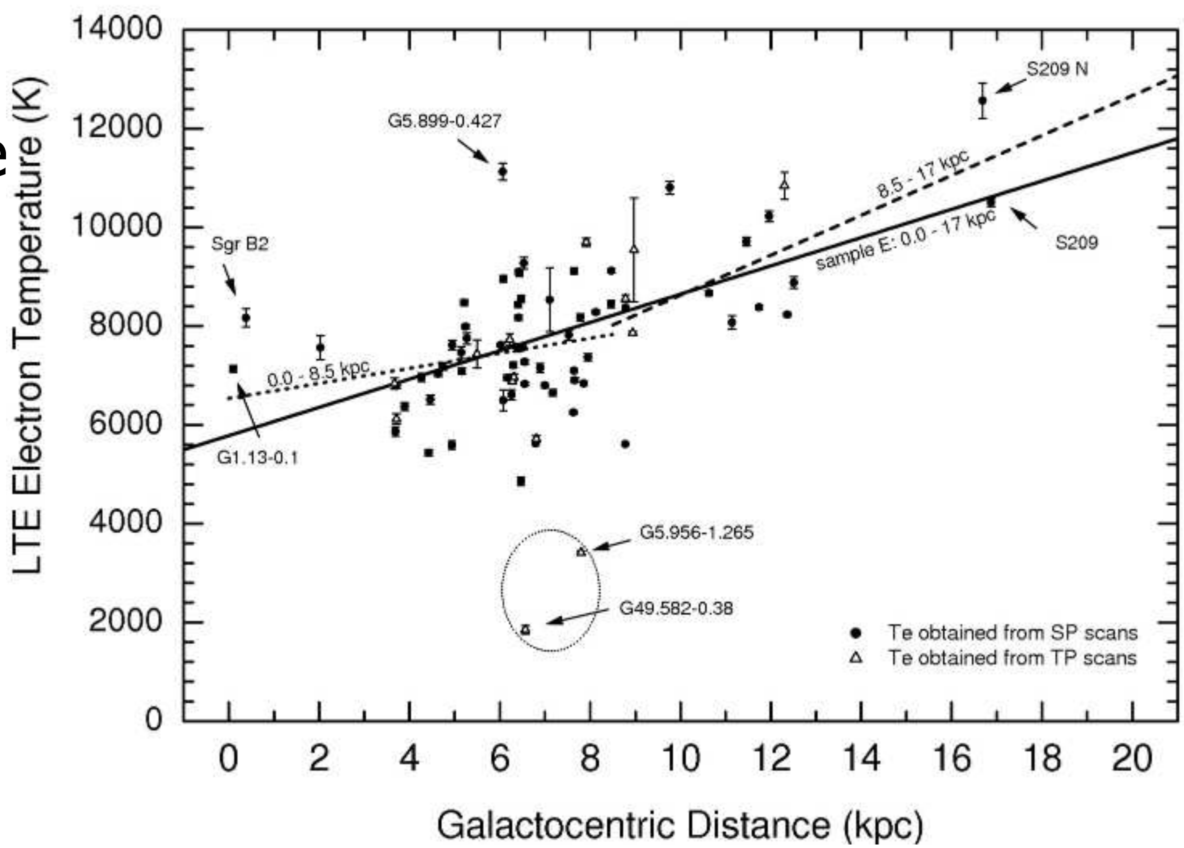
Spirals Arms of the Milky Way

- Traced by radio recombination lines (squares) and Ha (circles)
- Georgelin, Y. M., & Georgelin, Y. P. 1976, A&A, 49, 57

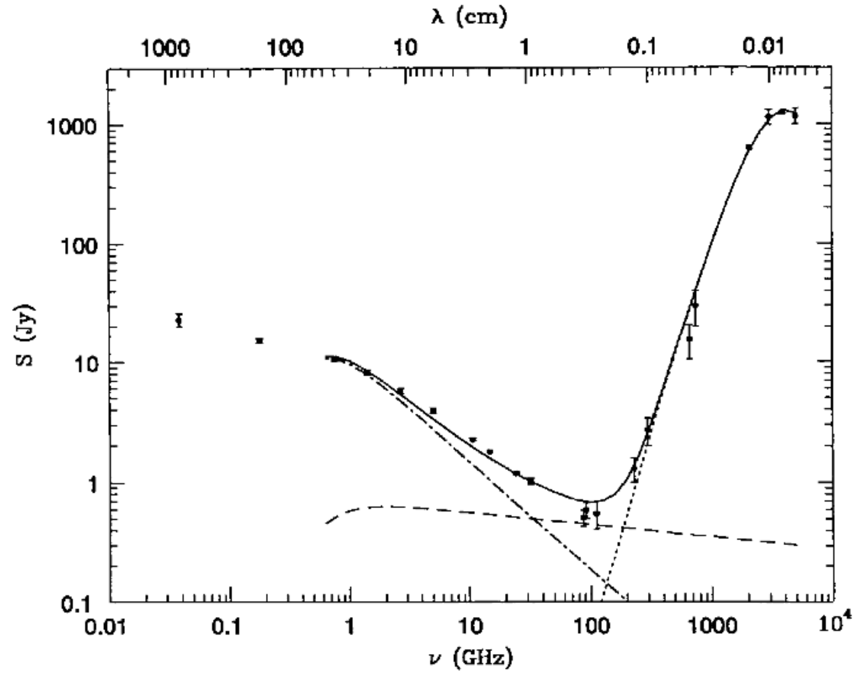


Radial Temperature/Metallicity Gradient in Milky Way

- Radio recombination line observations
- Quireza et al. 2006, ApJ, 653, 1226



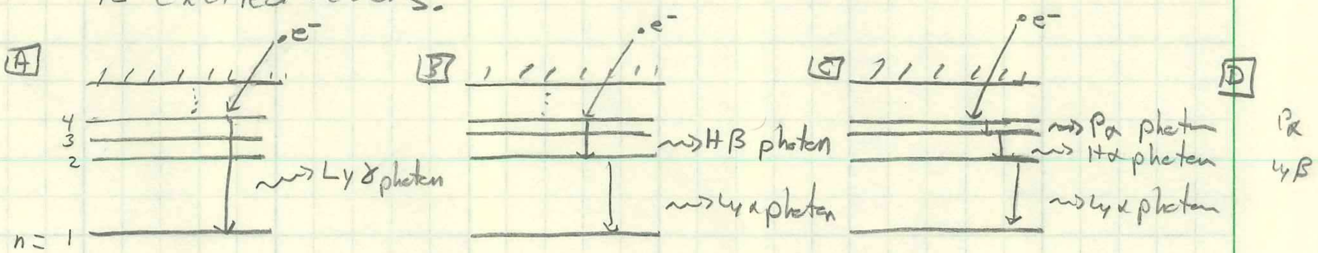
Starburst Galaxy M82 -- Template for high-redshift galaxies



- *Free-free emission shown by horizontal dashed line.*
- *Synchrotron radiation and thermal dust emission dominate at low and high frequencies*
- *Free-free absorption from HII regions distributed throughout the galaxy absorbs some of the synchrotron radiation and flattens the overall spectrum at the lowest frequencies.*

Recombination lines of H I, He I, and He II are characteristic features of spectra of gaseous nebulae.

Emitted by atoms undergoing radiative transitions in cascading down to the ground level following recombination to excited levels.



Recombination Cascade ⟹ Relative Strengths of Emission Lines

In a steady-state, statistical equilibrium requires

Rate Level nL Populated = Rate Level nL Depopulated

$$n_p n_e \alpha_{nL}(T) + \sum_{n'>n}^{\infty} \sum_{L'=L\pm1} n_{n'L'} A_{n'L', nL} + \sum_{L'=L\pm1} n_{nL'} n_p q_{nL', nL} =$$

Direct Recombination — b-b transitions from higher levels — Collisions that shift L within given n

$$\sum_{n'=1}^{n-1} \sum_{L'=L\pm1} n_{nL} A_{nL, n'L'} + \sum_{L'=L\pm1} n_{nL} n_p q_{nL, nL'}$$

bb-transitions to lower levels — Collisions that shift L within given n.

Note: Ignoring collisional excitation/de-excitation to/from ground state.

▷ When Important?
Hot e⁻s from hard ionizing spectrum. AGN!

See Osterbrock §11.8

Significant collisional cross sections involving excited levels of H

$$nL \rightarrow nL \pm 1$$

$$2^2S \rightarrow 2\,^2P$$

$$\sigma \sim 10^{-7} - 10^{-10}\ \text{cm}^2$$

$$^2L \rightarrow {}^2L \pm 1$$

Because of small energy difference between levels protons are more effective than e⁻s in causing collisions.

Limit of very low density –

ignore collisional terms between L states.

Table 4.2 $\alpha^{eff}_{H\beta} = 3.03 \times 10^{-14}\ cm^3 s^{-1}$ at 10^4 K no n_e

Table 4.4 $\alpha^{eff}_{H\beta} = 3.02 \times 10^{-14}\ cm^3 s^{-1}$ at $10^2\ cm^{-3}$

$= 3.03 \times 10^{-14}$ $10^4\ cm^{-3}$

$= 3.07 \times 10^{-14}$ $10^6\ cm^{-3}$

all at 10^4K

Demonstrate principle without including collisions –

Ⓐ statistical equilibrium

(1) $$n_p n_e \alpha_{nL}(T) + \sum_{n'>n}^{\infty} \sum_{L'=L''\pm 1} n_{n'L'} A_{n'L',nL} = \sum_{n'<n}^{n-1} \sum_{L'=L''\pm 1} n_{nL} A_{nL,n'L'}$$

Ⓑ level populations in thermodynamic equilibrium

Boltzmann Eqn. (Excitation) $$\frac{n'_{nL}}{n_{1s}} = (2L+1)\, e^{-\chi_n/kT}$$

Saha Eqn. (Ionization) $$\frac{n_p n_e}{n_{1s}} = \left(\frac{2\pi m k T}{h^2}\right)^{3/2} e^{-h\nu_0/kT}$$

$$\frac{n'_{nL}}{n_p n_e} = (2L+1)\left(\frac{h^2}{2\pi m k T}\right)^{3/2} e^{(h\nu_0 - \chi_n)/kT}$$ L.T.E.

Note: $h\nu_0 - \chi_n = h\nu_0 - \left(-\frac{h\nu_0}{n^2} + h\nu_0\right) = \frac{h\nu_0}{n^2}$

Find departure coefficients such that

$$N_{nL}(\text{steady state}) = b_{nL}(T_e)\, N'_{nL}(LTE)$$

T from Boltzmann or Saha eqn. closer to radiation temperature than electron temperature.

T (of level pop.) > T_e

Know b_n at high n.

? As $n \to \infty$, $b_n \to 1$. Think of e^-s getting close to continuum where $e^- - e^-$ collisions are Maxwell-Boltzmann.

Also - becomes more like Case A cause lines are optically thin.

Cut off equations at some highest n. (Low Population)

ⓒ Level m

- no transitions from $n > m$

$$n_p n_e \alpha_{mL} = \sum_{m'=0}^{m-1} \sum_{L'=L''\pm 1} n_{ml} A_{ml,n'L'}$$

$$n_{ml} = n_p n_e \frac{\alpha_{mL}}{\sum A_{ml,n'L'}}$$

$$b_m n'_{ml} = n_p n_e \frac{\alpha_{mL}}{\sum A_{ml,n'L'}}$$

$$b'_{ml} = \frac{1}{(2L+1)} \left(\frac{2\pi mkT}{h^2}\right)^{3/2} e^{-(h\nu_0/n^2)} \frac{\alpha_{mL}}{\sum A_{ml,n'L'}}$$

ⓓ Solve successively downward for b_{nl}.

Note: Departure coefficients b_{nl} become density dependent once collisions become important.

After the angular momentum changing transitions at fixed n $(\sigma_{10^2L} \to \sigma_{10^2L\pm1} \approx 4\times10^{-7}\,cm^{-2})$ the next largest collisional transition rates occur for collisions in which n changes by $\pm$, and of these the strongest are those for which L also changes by ± 1. (e^- collisions more effective than p collisions.)

Convenience - (Osterbrock p75) Use cascade matrix $C_{nL,n'L'}$ which is the probability that population of nL is followed by transition to $n'L'$ via all possible routes

$$n_p n_e \sum_{n'>n} \alpha_{n'L'} C_{n'L',nL} = \sum_{n'=1}^{n-1} \sum_{L'=L\pm1} n_{nL} A_{nL,n'L'}$$

Recaptures to higher levels that cascade to nL

Cascade matrix:
Probability that population on nL is followed by transition to $n'L'$ via all possible routes

$$P_{nL,n'L'} = \frac{A_{nL,n'L'}}{\sum_{n''=1}^{n-1} \sum_{L''} A_{nL,n''L''}} \qquad \text{note } A=0 \text{ unless } L'=L\pm 1$$

For $n'=n-1$

$$C_{nL,n-1L'} = P_{nL,n-1L'}$$

For $n'=n-2$

$$C_{nL,n-2L'} = P_{nL,n-2L'} + \sum_{L''=L'\pm 1} C_{nL,n-1L''} P_{n-1L'',n-2L'}$$

etc.

Define $C_{nL,nL''} = \delta_{LL''}$

Solutions to equilibrium eqns

$$n_p n_e \sum_{n'=n}^{\infty} \sum_{L'=0}^{n'-1} \alpha_{n'L'}(T)\, C_{n'L',nL} = n_{nL} \sum_{n''=1}^{n-1} \sum_{L''=L\pm 1} A_{nL,n''L'}$$

Population of any level nL fixed by balance of recombinations to all levels $n'\geq n$ that lead to cascades to nL and downward radiative transitions from nL

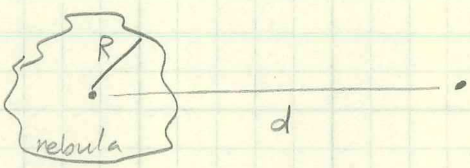
Easy to solve equation of statistical equilibrium in this form for either the level populations or departure coefficients.

Emission Coefficient — what you need to know!

Calculate emission coefficient once level populations found.

$$j_{nn'} = \frac{1}{4\pi} \sum_{l=0}^{n-1} n_{nl} A_{nl,n'l'} h\nu_{nn'} \overset{\text{CR}}{=} \frac{1}{4\pi} \sum b_{nl} n_{nl}^{LTE} A_{nl,n'l'} h\nu_{nn'}$$

zero unless $l' = l \pm 1$



$$L_{H\alpha} = h\nu_{H\alpha} \int_0^r n_p n_e \alpha^{eff}_{H\alpha}(H^e, T)\, dV$$

$$F_{nn'}(\text{at } d) = \frac{4\pi \int j_{nn'}\, dV}{4\pi d^2} = \frac{4\pi \int j_{nn'} r^2 dr}{d^2}$$

$$\Sigma_{nn'} = \frac{h\nu_{nn'} \int n_e n_p \alpha^{eff}_{nn'}\, ds}{4\pi} \qquad \frac{\text{energy}}{\text{time area}} \; \frac{1}{\text{steradian}}$$

Effective Recombination Coefficient

$$\alpha^{eff}_{nn'} = \frac{1}{n_p n_e} \frac{4\pi j_{nn'}}{h\nu_{nn'}} \qquad [\text{cm}^3\ \text{s}^{-1}]$$

analogous to total recombination coeff.

CR

$$j_{nn'} = \frac{1}{4\pi} n_p n_e \alpha^{eff}_{nn'} h\nu_{nn'} \qquad \text{ergs cm}^{-3}\ \text{sr}^{-1}$$

$$\alpha_{nn'} = b_n \frac{n_n(LTE)}{n_e n_p} \sum_{l=0}^{n-1} A_{nl,n'l'}$$

Application – Balmer Decrement

theory

equivalent

application

$$\frac{j_{n2}}{j_{H\beta}} = \frac{b_{n2}}{b_{H\beta}} \frac{A_{n2}}{A_{H\beta}} \frac{\nu_{n2}}{\nu_{H\beta}} \left[\frac{n_n^{LTE}}{n_4^{LTE}}\right]$$

see Osterbrock Table 4.1 and 4.2

$$= \frac{n_e n_p \alpha^{eff}_{n2} \nu_{n2}}{n_e n_p \alpha^{eff}_{H\beta} \nu_{H\beta}}$$

Case A:

$$\frac{\hat{j}_{H\alpha}}{\hat{j}_{H\beta}} = \begin{cases} 3.42 & T_e = 2500\ K \\ 3.10 & 5000 \\ 2.86 & 1e4 \\ 2.69 & 2e4 \end{cases}$$

Same temperature dependence.

$n_e \to 0$

$$\frac{\hat{j}_{Ly\alpha}}{\hat{j}_{H\beta}} = \begin{cases} 33.0 & T_e = 2500\ K \\ 32.5 & 5000 \\ 32.7 & 1e4 \\ 34.0 & 2e4 \end{cases}$$

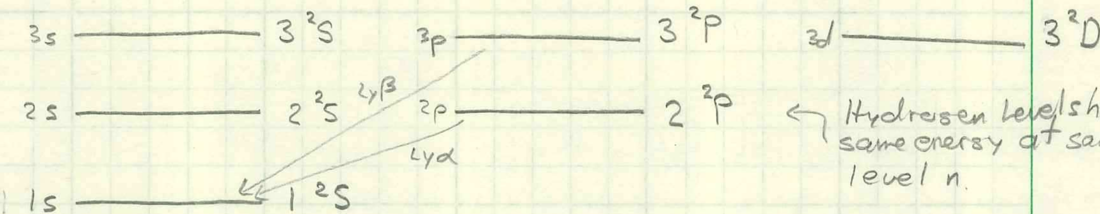
— good approximation for nebulae that are optically thin in all HI resonance lines

Case B:

Consider optical depth in center of a Lyman line

$\frac{1}{2} m_H v^2 = \pi kT$

$$\sigma_{P.I.}(Ly\,n) = \frac{3\lambda_{n1}^3}{8\pi}\left(\frac{m_H}{2\pi kT}\right)^{1/2} \underbrace{A_{nP,1S}}_{\text{note terms below}}$$



← Hydrogen levels have same energy at same level n.

What is $A_{Ly\alpha}$? →

At 10^4 K

$$\sigma_{P.I}(Ly\alpha) \sim \frac{3(1216\times10^{-8}\,cm)^3}{8\pi}\left[\frac{1.67\times10^{-24}}{2\pi(1.38\times10^{-16})(10^4 K)}\right]^{1/2}(10^8\ s^{-1})$$

$\sim 9.4\times10^{-15}\ cm^2$ $\qquad \frac{10^{-15}}{10}\ 10^{-6}\cdot10^{8} \sim 10^{-14}$

$\sim 10^{-14}\ cm^2$

Recall - $\sigma_{P.I.}(H,\nu_{L}) = 6.3\times10^{-18}\,cm^2 = 10^{-17}\,cm^2$

$\Rightarrow$ If you have a radiation bounded nebula it's probably optically thick in $Ly\alpha$! Resonant scattering

After several absorptions + re-emissions

$$\underset{4\to1}{Ly\gamma} \to \underset{4\to3}{Pa\alpha} + \underset{3\to2}{H\alpha} + Ly\alpha$$

$Ly\alpha \longrightarrow$ Dust absorption or 2γ continuum

Case B (cont'd):

Simply drop radiative $n^2P \to 1^2S$ transitions from statistical equilibrium equations.

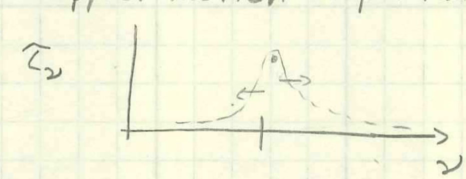
Osterbrock - Table 4.2 | Table 4.4 — changes by a few %.

$$\frac{j_{H\alpha}}{j_{H\beta}} =$$

Table 4.2	T_e	Table 4.4
3.30	$T_e = 2500$ K	—
3.05	5000	3.04
2.87	10,000	$\boxed{2.86}$
2.76	20,000 K	2.75

$n_e = 10^2\ cm^{-3}$

Note: Doppler motion important for Lyα emission



see Brocklehurst Lyα

Photon random walks in frequency space until it scatters out of line profile.

Case C: (see Osterbrock §11.8 for details)

Regions photoionized by high energy photons — power law UV spectrum.

$\boxed{At\ T_e > 1.25 \times 10^4 K}$ collisional excitation of Lyα and recombination populate $n=2$ level of H.

$$H^\circ(1s\ ^2S) + e^- \longrightarrow H^\circ(2p\ ^2P) + e^-$$

lower energy

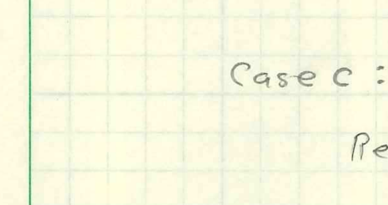
Affects other line ratios too!

Consider collisional excitation to $3\,^2S, 3\,^2P, 3\,^2D$

$\Rightarrow$ Hα emission

Collisional-excitation emission coeff.

$4\pi j_{H\alpha} = n_e n_p \sum' q_{1^2S, 3^2L}\, h\nu_{H\alpha}$



$\Rightarrow$ intrinsic Balmer Decrement

$\frac{H\alpha}{H\beta} = 3.1$ observed for AGN NLR

vs $= 2.86$ the recombination value

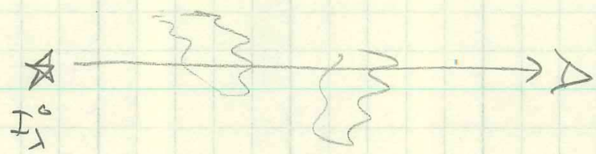
See Also: §4.5 Osterbrock Between case A and case B Lyα - CCX 1969 ApJ 155, 859

• Balmer Decrement and Interstellar Extinction

HII Regions -

Generally measure $\frac{F_{H\alpha}}{F_{H\beta}} < 2.86$.

Measurement used to correct for effects of dust



$$I_\lambda = I_\lambda^0 e^{-\tau_\lambda}$$

works for absorption or scattering out of the beam.

Note:

Dust + stars mixed → scattering into beam. (see Osterbrock §7.3)

$$\tau_\lambda = c\, f(\lambda)$$

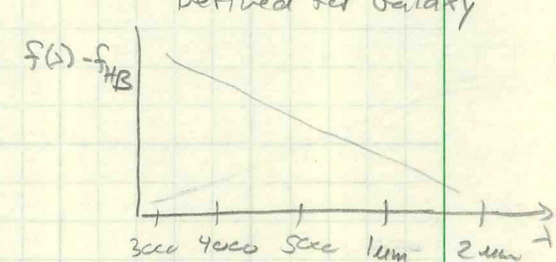
$f(\lambda)$ ← same for all stars

c ← Depends on path to star.

Osterbrock Table 7.2

λ (Å)	$f(\lambda) - f(H\beta)$
$H\alpha$	-0.37
$H\beta$	0
$H\gamma$	0.13
$Ly\alpha$	1.93

Derived for Galaxy



Use relative line intensities and solve for c

Most Common:

$$\frac{I_{H\alpha}}{I_{H\beta}} = \frac{I_{H\alpha}^0}{I_{H\beta}^0} \frac{e^{-cf(H\alpha)}}{e^{-cf(H\beta)}} = \frac{I_{H\alpha}^0}{I_{H\beta}^0} e^{-c(f(H\alpha) - f(H\beta))}$$

$f(H\alpha) - f(H\beta) = -0.37 - 0 = -0.37$

Beware: Definitions of c and f vary!

Better:

Use lines with same upper state; e.g. $P\alpha$ $4 \to 3$ (1.87μm), $H\beta$ $4 \to 2$

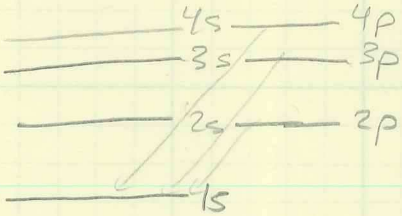
Best:

Extrapolate to $\lambda \to \infty$

Compare intensity of thermal radio continuum to optical HI line.

Case B Recombination Line Spectrum of Hydrogen

Optically thick to radiation just above $I_H = 13.60$ eV.
Recombinations to $n=1$ do not reduce the ionization of the gas.



Lyman series $\rightarrow$ allowed transitions with large $A_{np,1s}$ values.

The optical depth in a Lyman line is large.

$$\sigma_{Abs}(Ly\,n) = \frac{3\,\lambda_{n1}^3}{8\pi}\left(\frac{m_H}{2\pi kT}\right)^{1/2} A_{np,1s}$$

At 10^4 K,

$$\sigma_{Abs}(Ly\alpha) = \frac{3\,(1216\times10^{-8}\,\mathrm{cm})^3}{8\pi}\left(\frac{1.67\times10^{-24}\,\mathrm{g}}{2\pi\,(1.38\times10^{-16}\,\mathrm{ergs/K})(10^4\,\mathrm{K})}\right)^{1/2} A_{2p,1s}$$

$$\simeq 9.4\times10^{-15}\,\mathrm{cm}^2\left[\frac{A_{2p,1s}}{10^8\,\mathrm{s}^{-1}}\right]$$

Compare $\sigma_{PI}(H,\nu_0) = 6.3\times10^{-18}\,\mathrm{cm}^2$ <u>About 1000 x lower!</u>

The cross-sections for absorption in the Lyman series is a decreasing function of n, as $n\to\infty$ this cross section becomes equal to the photoionization cross section near ν_0.

$\Longrightarrow$ All the <u>Lyman series</u> transitions are <u>optically thick</u> for Case B conditions.

- Lyman photons emitted will be resonantly absorbed by other H atoms in the groundstate (nearby).
- The excited np level immediately decays

$np \rightarrow 1s$ just scatters the Ly n photon again

$np \rightarrow n'd$, $n's$ where $n' < n$, producing a non-Lyman photon which escapes.

When $n \geq 3$ there is always an escape channel.
Hence no Lyman lines emerge.

For Case B emissivities, we use

$$4\pi j(nl \to n'l') = n_e n(H^+) \frac{A(nl \to n'l')\, h\nu_{ul}}{\sum'_{n''l''} A(nl \to n''l'')} \times \left[\alpha(nl) + \sum'_{n''l'';\, n''>n} \alpha(n''l'')\, P_A(n''l'', nl) \right]$$

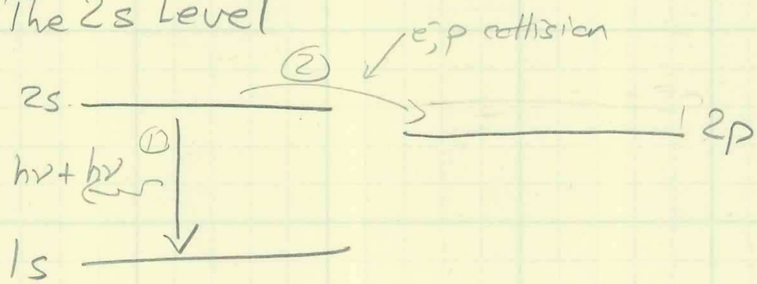
but replace the $A_{np \to 1s}$ rates for $n \geq 3$ with $\emptyset$ and with the probabilities P_A replaced by probabilities P_B calculated with the Lyman series transition probabilities set to $\emptyset$.

- The recombination cascade must eventually take the atom to either the 2s level or the 2p level.

$$\Longrightarrow \quad \alpha_B = \underset{(1/3)}{\alpha_{eff,2s}} + \underset{(2/3)}{\alpha_{eff,2p}}$$

what happens to the e^- next?

1) The 2s Level



① $A(2s \to 1s) = 8.23\ s^{-1}$ Highly Forbidden
Continuous spectrum from $\nu = 0$ to $\nu(Ly\alpha)$ symmetric about $\nu = \nu_{Ly\alpha}/2$.

② Collisional deexcitation rate 2s to 2p is much larger than for 2s to 1s. At $n_e > n_{crit} \simeq 1880\ cm^{-3}$, the <u>2-photon continuum</u> is <u>suppressed by collisional processes</u>.

2) The 2p level quickly decays radiatively,

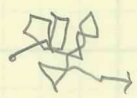
$A(2p \to 1s) = 6.3 \times 10^8 \, s^{-1}$.

No time for collisional deexcitation at ISM densities.

$$\tau_0(Ly\alpha) = 8.02 \times 10^4 \left(\frac{15 \, km\, s^{-1}}{b}\right) \tau(Ly \text{ cont}),$$

where $\tau(Ly \text{ cont}) = N(H)\, 6.30 \times 10^{-18} \, cm^{-2}$.

$\Rightarrow$ So $\tau(Ly\alpha) \geq 10^5$ under case B conditions, and the photons scatter <u>many</u> times.



Random walk in space and frequency

→ Absorbed by dust grains

→ Escape after diffusing into the line wings due to random velocities of H atoms.

<u>Case B Ratios for $Ly\alpha$:</u>

$\alpha_{eff,2P}/\alpha_B$?

Rate of $Ly\alpha$ photon production $= f(2p) \cdot f_{ion}\, Q_0$

$= \frac{2}{3} \cdot f_{ion} \cdot Q_0$

CCX 1969 ApJ 155 859

Rate of $H\alpha$ photon production $= \frac{L_{H\alpha}}{h\nu_{H\alpha}} = \frac{j_{H\alpha} \cdot V}{h\nu_\alpha} = n_p n_e \alpha_{eff,H\alpha} \cdot V$

Brocklehurst

$$= \frac{Q(H^0)}{\alpha_B} \alpha_{eff,H\alpha}$$

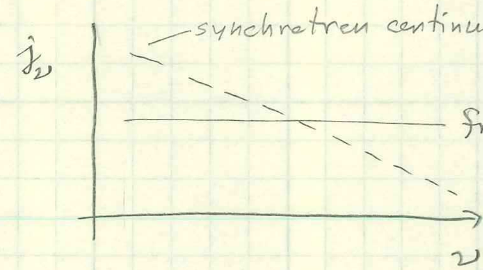
$$\frac{L(Ly\alpha)}{L(H\alpha)} = \frac{h\nu_{Ly\alpha}}{h\nu_{H\alpha}} \frac{2}{3} \frac{Q(H^0)}{Q(H^0)} \frac{\alpha_B}{\alpha_{eff,H\alpha}} f_{ion}$$

$$= \frac{6563}{1216} \cdot \frac{2}{3} \frac{2.59 \times 10^{-13}}{1.17 \times 10^{-13}} \quad \text{at } T_e = 10^4 \, K$$

$$= 7.97$$

• Radio Spectra of H II Regions

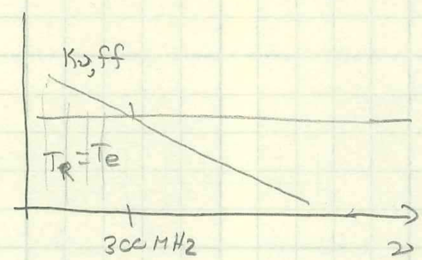
+ Reality – separation between thermal emission and nonthermal synchrotron radiation is problem.



$\Rightarrow$ assess thermal contribution at high frequencies

e.g. Beck + Turner
NGC5253 – purely thermal

+ H II Regions are optically thick at low frequencies (1m, $\nu < 300$ MHz)



PROBLEM: Beam size
See Osterbrock 5.4

Measured intensity J_ν (or brightness temperature) is the temperature within the nebula

$T_{b\nu} = T(1-e^{-\tau_\nu}) \Rightarrow T$ as $\tau_\nu \to \infty$

+ Free-free emissivity depends on n_e^2 in same way as recombination lines. $I_{mn} = \int I_\nu d\nu = h\nu_{mn}\, \alpha_{mn} \frac{n_p}{n_e} \int n_e^2 ds$

Determine mean e^- density from Emission Measure.

$$EM \equiv \int_0^L n_e^2\, ds \qquad cm^{-6}\ pc$$

RMS electron density

(1) H II Regions: $EM \sim 10^3$ pc cm^{-6}

For $2R_S \sim 10$ pc

$$\langle n_e^2 \rangle^{1/2} \sim \sqrt{10^3/10} = 10\ cm^{-3}$$

Note: Mean is much less than local density measured from collisionally excited lines $\Rightarrow n_e \sim 10^2\ cm^{-3}$.

Define volume filling factor

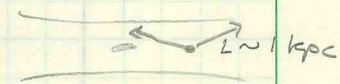
$\langle n_{rms}^2 \rangle = \epsilon\, n_e^2 \qquad \Rightarrow \epsilon = \frac{(10^2)}{(10^2)^2} = 10^{-2} \qquad$ 1% full

Emission Measure (cont'd)

② Diffuse Galactic Emission

WHAM + Heiles

$$EM \sim 5-15 \quad pc\ cm^{-6}$$



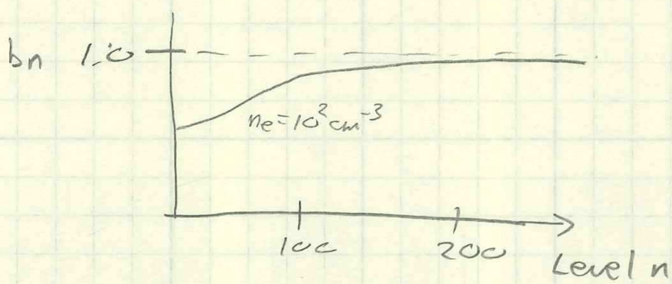
$$n_{rms} = \langle n_e^2 \rangle^{1/2} = \left[\frac{10\ pc\ cm^{-6}}{10^3\ pc}\right]^{1/2} = 10^{-1} = 0.1\ cm^{-3}$$

$$P_{WIM} \sim k n_e T \sim 0.1\ cm^{-3}\ (10^4\ K)\ k$$

$$\frac{P}{k} \sim 10^3\ K\ cm^{-3}$$ Typical Interstellar Pressure in Diffuse Phase

+ Also observe radio recombination lines

show example ? Heiles



osterbrock Fig. 4.2

Hydrogen Departure Coeff

Note: Increasing importance of collisional transitions.
$b_n \to 1$ as $n \to \infty$

To calculate emission in specific line

① solve eqn. of transfer

② use line absorption coefficient corrected for stimulated emission.

$$\kappa_{\nu L} = \kappa_{\nu l}\left(1 - \frac{b_m}{b_n} e^{-h\nu/kT}\right)$$

negative ⇒ maser action

Note: Ratio of brightness temperature in line and continuum gives ratio of optical depths which measures T.
- Not that great. Deviations from LTE matter.
osterbrock § 5.7