

UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Physics 233

Study Guide

Winter 2014

We have covered the following material: **Draine:** ch 1 - 10, 13 - 19, 21 - 24, 27, 29 - 32, 34 and **Osterbrock:** ch 1 - 5. I recommend you look over the section headers and figures and review all homework problems. I hope you take away from this class an understanding of the following concepts:

- Why diffuse plasmas are generally not in local thermodynamic equilibrium.
- What the spectrum of a diffuse plasma look like.
- What the principle cooling processes are in a diffuse gas. [Note, the answer depends on its temperature.]
- How the level populations determined in a collisionally excited gas. [Know the law of mass action, the principle of detailed balance, and the definition of the critical density.
- How the level populations determined in a photoionized gas.
- The equations that describe photoionization equilibrium, the derivation of the Stromgren radius, the conditions required for Case B or Case A recombination, and the definition of the ionization parameter.
- How we calculate the equilibrium temperature of each of the following: HII regions, HI regions, molecular clouds, and dust grains.
- Know what determines the Energy levels in atoms, ions, and molecules, the rules for bound-bound transitions, and the processes describing bound-free and free-bound transitions.
- Be able to write down the Equation of radiative transfer and solve it for simple cases. Understand the difference between specific intensity, flux, and radiation pressure.
- Be familiar with the observed properties of interstellar dust and the inferred sizes and composition of the grains. Know how to apply corrections for reddening and extinction to observed spectra.
- Know what factors determine whether the interstellar medium in a galaxy can support multiple gas phases.
- Know how molecular hydrogen is formed and destroyed.
- Know why (and how) the CO molecular can be used to estimate the molecular gas mass in galaxies.

- Know when (and how) spectral lines from molecules can be used as a thermometer to estimate the gas temperature.

Physical Constants and Astronomical Data

New! Try my [Physical Calculator](#). It is a JavaScript calculator with all of the constants below programmed into it.

Physical Constants

(converted to CGS units from the [NIST Constant Index](#))

| Name | Symbol Number | Exp CGS Units | Relative Error (ppm) |
|----------------------------|----------------|---|----------------------|
| speed of light in a vacuum | c | 2.99792458 10 cm s ⁻¹ | exact |
| Planck constant | h | 6.6260755(40) -27 erg s | 0.60 |
| | ħ | 1.05457266(63) -27 erg s | 0.60 |
| Gravitational constant | G | 6.67259(85) -8 cm ³ g ⁻¹ s ⁻² | 128 |
| Electron charge | e | 4.8032068(14) -10 esu | 0.30 |
| Mass of electron | m _e | 9.1093897(54) -28 g | 0.59 |
| Mass of proton | m _p | 1.6726231(10) -24 g | 0.59 |
| Mass of neutron | m _n | 1.6749286(10) -24 g | 0.59 |
| Mass of hydrogen | m _H | 1.6733 -24 g | -- |
| Atomic mass unit | amu | 1.6605402(10) -24 g | 0.59 |
| Avagadro's number | N _A | 6.0221367(36) 23 | 0.59 |
| Boltzmann constant | k | 1.380658(12) -16 erg k ⁻¹ | 8.5 |
| Electron volt | eV | 1.6021772(50) -12 erg | ~0.60 |
| Radiation density constant | a | 7.5646 -15 erg cm ⁻³ K ⁻⁴ | -- |
| Stefan-Boltzmann constant | σ | 5.67051(19) -5 erg cm ⁻² K ⁻⁴ s ⁻¹ | 34 |
| Fine structure constant | α | 7.29735308(33) -3 | 0.045 |
| Rydberg constant | R _∞ | 2.1798741(13) -11 erg | 0.60 |

Note: a "--" in the error column means that I have not found a good source for that constant, so the value quoted is just an approximation

Astronomical Units/Data

| NAME | SYMBOL | NUMBER | EXP | CGS UNITS |
|-------------------|-------------|--------|-----|---------------------|
| Astronomical unit | AU | 1.496 | 13 | cm |
| Parsec | pc | 3.086 | 18 | cm |
| Light year | ly | 9.463 | 17 | cm |
| Solar mass | M_{\odot} | 1.99 | 33 | g |
| Solar radius | R_{\odot} | 6.96 | 10 | cm |
| Solar luminosity | L_{\odot} | 3.9 | 33 | erg s ⁻¹ |
| Solar Temperature | T_{\odot} | 5.780 | 3 | K |

| NAME | MASS (g) | RADIUS (cm) | PERIOD (Yr) | SEMI-MAJOR (AU) | ECCENTRICITY | |
|---------|----------|-------------|-------------|-----------------|--------------|----------|
| Mercury | 3.303 26 | 2.439 8 | 2.4085 | -1 | 3.87096 -1 | 0.205622 |
| Venus | 4.870 27 | 6.050 8 | 6.1521 | -1 | 7.23342 -1 | 0.006783 |
| Earth | 5.976 27 | 6.378 8 | 1.00004 | 0 | 9.99987 -1 | 0.016684 |
| Mars | 6.418 26 | 3.397 8 | 1.88089 | 0 | 1.523705 0 | 0.093404 |
| Jupiter | 1.899 30 | 7.140 9 | 1.18622 | 1 | 5.204529 0 | 0.047826 |
| Saturn | 5.686 29 | 6.000 9 | 2.94577 | 1 | 9.575133 0 | 0.052754 |
| Uranus | 8.66 28 | 2.615 9 | 8.40139 | 1 | 1.930375 1 | 0.050363 |
| Neptune | 1.030 29 | 2.43 9 | 1.64793 | 2 | 3.020652 1 | 0.004014 |
| Pluto | 1. 25 | 1.2 8 | 2.47686 | 2 | 3.991136 1 | 0.256695 |

A Few Conversion Factors

| CGS | --> | SI | Multiply by |
|-----------------|-----|---------------|------------------|
| centimeter (cm) | | meter (m) | 10 ⁻² |
| gram (g) | | kilogram (kg) | 10 ⁻³ |
| erg | | Joule (J) | 10 ⁻⁷ |
| dyne (dyn) | | newton (N) | 10 ⁻⁵ |

IONIZATION POTENTIALS^a

Spectrum

| Z | Element | I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII | XIII | XIV | XV | XVI | XVII | XVIII | XIX | XX | XXI | |
|----|---------|--------|--------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-------|-----|-------|------|--|
| 1 | H | 13.598 | | | | | | | | | | | | | | | | | | | | | |
| 2 | He | 24.587 | 54.416 | | | | | | | | | | | | | | | | | | | | |
| 3 | Li | 5.392 | 75.638 | 122.451 | | | | | | | | | | | | | | | | | | | |
| 4 | Be | 9.322 | 18.211 | 153.893 | 217.713 | | | | | | | | | | | | | | | | | | |
| 5 | B | 8.298 | 25.154 | 37.990 | 259.368 | 340.217 | | | | | | | | | | | | | | | | | |
| 6 | C | 11.260 | 24.383 | 47.887 | 64.492 | 392.077 | 489.981 | | | | | | | | | | | | | | | | |
| 7 | N | 14.534 | 29.601 | 47.448 | 77.472 | 97.888 | 552.057 | 667.029 | | | | | | | | | | | | | | | |
| 8 | O | 13.618 | 35.116 | 54.934 | 77.412 | 113.896 | 138.116 | 739.315 | 871.387 | | | | | | | | | | | | | | |
| 9 | F | 17.422 | 34.970 | 62.707 | 87.138 | 114.240 | 157.161 | 185.182 | 953.886 | 1103.089 | | | | | | | | | | | | | |
| 10 | Ne | 21.564 | 40.962 | 63.45 | 97.11 | 126.21 | 157.93 | 207.27 | 239.09 | 1195.797 | 1362.164 | | | | | | | | | | | | |
| 11 | Na | 5.139 | 47.286 | 71.64 | 98.91 | 138.39 | 172.15 | 208.47 | 264.18 | 299.87 | 1465.091 | 1648.659 | | | | | | | | | | | |
| 12 | Mg | 7.646 | 15.035 | 80.143 | 109.24 | 141.26 | 186.50 | 224.94 | 265.90 | 327.95 | 367.53 | 1761.802 | 2304.080 | | | | | | | | | | |
| 13 | Al | 5.986 | 18.828 | 28.447 | 45.141 | 66.777 | 205.05 | 246.52 | 303.17 | 351.10 | 401.43 | 476.06 | 523.50 | 2437.676 | 2673.108 | | | | | | | | |
| 14 | Si | 8.151 | 16.345 | 33.492 | 51.37 | 65.023 | 230.43 | 263.22 | 309.41 | 371.73 | 424.50 | 479.57 | 560.41 | 611.85 | 2816.943 | 3069.762 | | | | | | | |
| 15 | P | 10.486 | 19.725 | 30.18 | 47.30 | 72.68 | 88.049 | 280.93 | 328.23 | 379.10 | 447.09 | 504.78 | 564.65 | 651.63 | 707.14 | 809.39 | 3658.425 | 3946.193 | | | | | |
| 16 | S | 10.360 | 23.33 | 34.83 | 53.46 | 67.8 | 98.03 | 114.193 | 348.28 | 400.05 | 455.62 | 529.26 | 591.97 | 656.69 | 749.74 | 854.75 | 918 | 1034 | 1087 | | | | |
| 17 | Cl | 12.967 | 23.81 | 39.61 | 59.81 | 75.02 | 91.007 | 124.319 | 143.456 | 422.44 | 478.68 | 538.95 | 618.24 | 686.09 | 755.73 | 854.75 | 918 | 1034 | 1087 | | | | |
| 18 | Ar | 15.759 | 27.629 | 40.74 | 59.81 | 75.02 | 91.007 | 124.319 | 143.456 | 422.44 | 478.68 | 538.95 | 618.24 | 686.09 | 755.73 | 854.75 | 918 | 1034 | 1087 | | | | |
| 19 | K | 4.341 | 31.625 | 45.72 | 60.91 | 82.66 | 100.0 | 117.56 | 154.86 | 175.814 | 503.44 | 564.13 | 629.09 | 714.02 | 787.13 | 861.77 | 968 | 1034 | 1087 | | | | |
| 20 | Ca | 6.113 | 11.871 | 50.908 | 67.10 | 84.41 | 108.78 | 127.7 | 147.24 | 188.54 | 211.270 | 249.832 | 291.497 | 326.03 | 416.61 | 489.12 | 546.8 | 571 | 607.2 | 633 | 671 | 1698 | |
| 21 | Sc | 6.54 | 12.80 | 24.76 | 73.47 | 91.66 | 111.1 | 138.0 | 158.7 | 180.02 | 225.32 | 249.832 | 291.497 | 326.03 | 416.61 | 489.12 | 546.8 | 571 | 607.2 | 633 | 671 | 1698 | |
| 22 | Ti | 6.82 | 13.58 | 27.491 | 43.266 | 99.22 | 119.36 | 140.8 | 168.5 | 193.2 | 215.91 | 265.23 | 291.497 | 326.03 | 416.61 | 489.12 | 546.8 | 571 | 607.2 | 633 | 671 | 1698 | |
| 23 | V | 6.74 | 14.65 | 29.310 | 46.707 | 65.23 | 128.12 | 150.17 | 173.7 | 205.8 | 230.5 | 255.04 | 308.25 | 336.267 | 384.30 | 404 | 435.3 | 457 | 489.5 | 512 | 546.8 | 571 | |
| 24 | Cr | 6.766 | 16.50 | 30.96 | 49.1 | 69.3 | 90.56 | 161.1 | 184.7 | 209.3 | 244.4 | 270.8 | 298.0 | 355 | 384.30 | 404 | 435.3 | 457 | 489.5 | 512 | 546.8 | 571 | |
| 25 | Mn | 7.435 | 15.640 | 33.667 | 51.2 | 72.4 | 95 | 119.27 | 196.46 | 221.8 | 248.3 | 286.0 | 314.4 | 343.6 | 384.30 | 404 | 435.3 | 457 | 489.5 | 512 | 546.8 | 571 | |
| 26 | Fe | 7.870 | 16.18 | 30.651 | 54.8 | 75.0 | 99 | 125 | 151.06 | 235.04 | 262.1 | 290.4 | 330.8 | 361.0 | 392.2 | 457 | 489.5 | 512 | 546.8 | 571 | 607.2 | 633 | |
| 27 | Co | 7.86 | 17.06 | 33.50 | 51.3 | 79.5 | 102 | 129 | 157 | 186.13 | 276 | 305 | 336 | 379 | 411 | 444 | 499 | 571 | 607.2 | 633 | 671 | 1698 | |
| 28 | Ni | 7.635 | 18.168 | 35.17 | 54.9 | 75.5 | 108 | 133 | 162 | 193 | 224.5 | 321.2 | 352 | 384 | 430 | 464 | 499 | 571 | 607.2 | 633 | 671 | 1698 | |
| 29 | Cu | 7.726 | 20.292 | 36.83 | 55.2 | 79.9 | 103 | 139 | 166 | 199 | 232 | 266 | 368.8 | 401 | 435 | 484 | 520 | 557 | 619 | 698 | 738 | 1856 | |
| 30 | Zn | 9.394 | 17.964 | 39.722 | 59.4 | 82.6 | 108 | 134 | 174 | 203 | 238 | 274 | 310.8 | 419.7 | 454 | 490 | 542 | 579 | 619 | 698 | 738 | 1856 | |
| 31 | Ga | 5.999 | 20.51 | 30.71 | 64 | | | | | | | | | | | | | | | | | | |
| 32 | Ge | 7.899 | 15.934 | 34.22 | 45.71 | 93.5 | | | | | | | | | | | | | | | | | |
| 33 | As | 9.81 | 18.633 | 28.351 | 50.13 | 62.63 | 127.6 | | | | | | | | | | | | | | | | |
| 34 | Se | 9.752 | 21.19 | 30.820 | 42.944 | 68.3 | 81.70 | 155.4 | | | | | | | | | | | | | | | |
| 35 | Br | 11.814 | 21.8 | 36 | 47.3 | 59.7 | 88.6 | 103.0 | 192.8 | | | | | | | | | | | | | | |
| 36 | Kr | 13.999 | 24.359 | 36.95 | 52.5 | 64.7 | 78.5 | 111.0 | 126 | 230.39 | | | | | | | | | | | | | |
| 37 | Rb | 4.177 | 27.28 | 40 | 52.6 | 71.0 | 84.4 | 99.2 | 136 | 150 | 277.1 | | | | | | | | | | | | |
| 38 | Sr | 5.695 | 11.030 | 43.6 | 57 | 71.6 | 90.8 | 106 | 122.3 | 162 | 177 | 324.1 | | | | | | | | | | | |
| 39 | Y | 6.38 | 12.24 | 20.52 | 61.8 | 77.0 | 93.0 | 116 | 129 | 146.52 | 191 | 206 | 374.0 | | | | | | | | | | |
| 40 | Zr | 6.84 | 13.13 | 22.99 | 34.34 | 81.5 | | | | | | | | | | | | | | | | | |
| 41 | Nb | 6.88 | 14.32 | 25.04 | 38.3 | 50.55 | 102.6 | 125 | | | | | | | | | | | | | | | |
| 42 | Mo | 7.099 | 16.15 | 27.16 | 46.4 | 61.2 | 68 | 126.8 | 153 | | | | | | | | | | | | | | |
| 43 | Te | 7.28 | 15.26 | 29.54 | | | | | | | | | | | | | | | | | | | |
| 44 | Ru | 7.37 | 16.76 | 28.47 | | | | | | | | | | | | | | | | | | | |
| 45 | Rh | 7.46 | 18.08 | 31.06 | | | | | | | | | | | | | | | | | | | |
| 46 | Pd | 8.34 | 19.43 | 32.93 | | | | | | | | | | | | | | | | | | | |
| 47 | Ag | 7.576 | 21.49 | 34.83 | | | | | | | | | | | | | | | | | | | |

Planck Function: $B_{\nu}(T) = \frac{2h\nu^3}{c^2} \left[\frac{1}{e^{h\nu/kT} - 1} \right]$

Wien's Law: $\lambda_{\max} T = 0.29 \text{ cm} \cdot \text{K}$

Maxwell-Boltzmann Distribution: $f(v) = \frac{4}{\sqrt{\pi}} \left(\frac{m}{2kT} \right)^{3/2} v^2 e^{-mv^2/2kT}$

$$f(E) = \frac{2N}{\sqrt{\pi} (kT)^{3/2}} E^{1/2} e^{-E/kT} dE$$

Saha Equation: $\frac{n_p n_e}{n_{1s}} = \left(\frac{2\pi m kT}{h^2} \right)^{3/2} e^{-h\nu_0/kT}$

Boltzmann Equation: $\frac{N_j}{N_i} = \frac{g_j}{g_i} e^{-\Delta E/kT}$, where $g_j = 2J+1$

1.

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

2.

$$\int x e^{ax} dx = \frac{e^{ax}}{a} \left(x - \frac{1}{a} \right)$$

3.

$$\int x^2 e^{ax} dx = \frac{e^{ax}}{a} \left(x^2 - \frac{2x}{a} + \frac{2}{a^2} \right)$$

4.

$$\begin{aligned} \int x^n e^{ax} dx &= \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \\ &= \frac{e^{ax}}{a} \left(x^n - \frac{nx^{n-1}}{a} + \frac{n(n-1)x^{n-2}}{a^2} - \dots - \frac{(-1)^n n!}{a^n} \right) \end{aligned}$$

5.
$$\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

6.
$$\int_0^{\infty} e^{-ax^2} \cos bx dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-b^2/4a}$$

7.
$$\int_{-b/2a}^{\infty} e^{-(ax^2+bx+c)} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{(b^2-4ac)/4a}$$

8.
$$\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)} dx = \sqrt{\frac{\pi}{a}} e^{(b^2-4ac)/4a}$$

9.
$$\int_0^{\infty} x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{(n+1)}}$$

10.
$$\int_0^{\infty} x^m e^{-ax^2} dx = \frac{\Gamma[(m+1)/2]}{2a^{(m+1)/2}}$$

11.
$$\int_0^{\infty} e^{-(ax^2+b/x^2)} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{-2\sqrt{ab}}$$

12.
$$\int_0^{\infty} \frac{xdx}{e^x - 1} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$$