University of California, Santa Barbara
Department of Physics

PHYSICS 24	FINAL EXAM	WINTER 2004
A. N. Cleland	8-11 am	Tuesday, March 16, 2004

- **This exam is closed book, closed notes.**
- **Calculators are allowed**
- **Show all work clearly. Partial credit will be given if your thinking is clear.**

Write your name:

KEY

Scoring:

Problem 1	____	of 20
Problem 2	____	of 20
Problem 3	____	of 20
Problem 4	____	of 20
Problem 5	____	of 20
Problem 6	____	of 20
Total	____	of 120

Problem 1 (20 points). An accelerating charge radiates electromagnetic power, at a rate

$$\frac{dE}{dt} = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}$$

where q is the charge, a is the acceleration, $c = 3 \times 10^8$ m/s the speed of light and $\varepsilon_0 = 8.85 \times 10^{-12}$ C/m the permittivity of vacuum.

If an electron ($q = 1.6 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) has a kinetic energy of 6.0 ~~M~~ k eV, and is traveling in an orbit with radius 0.5 m, what fraction of its energy does it radiate per second (assume constant speed)?

5 $a = \frac{v^2}{R}$ $\quad \frac{1}{2} m v^2 = 6\,keV = 6 \cdot 10^3 \cdot 1.6 \cdot 10^{-19}\,J \Rightarrow v^2 = 2.1 \cdot 10^{15}\,\frac{m^2}{s^2}$

$R = 0.5\,m$

5 $a = 4.2 \cdot 10^{15}\,m/s^2$

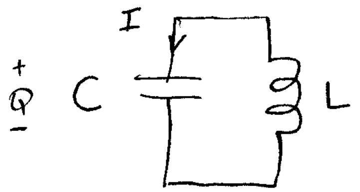
5 $\frac{dE}{dt} = 1.01 \cdot 10^{-22}\,J/s = 6.3 \cdot 10^{-4}\,\frac{eV}{s}$ 2

$\boxed{\frac{1}{E}\frac{dE}{dt} = 10^{-7}/sec.}$ 3

Problem 2 (20 points) An *L-C* circuit (no resistance) has its capacitor at maximum charge at time $t = 0$.

(a) If *T* is the period of oscillation of the circuit, find at the time at which the electric and magnetic energies will be *equal* in the circuit, in terms of *T*.

(b) How much longer after the time in (a) must you wait for the energies to again be equal? Again, give your answer in terms of *T*.



$$L\frac{dI}{dt} = \frac{Q}{C}$$

$$L\ddot{I} - \frac{I}{C} = 0$$

$$T = 2\pi\sqrt{LC} = \frac{2\pi}{\omega}$$

$$\omega = \frac{1}{\sqrt{LC}} \quad \phi = \pi$$

5
$$\begin{cases} I = +I_0 \sin \omega t \\ Q = Q_0 \cos(\omega t) : \; @t=0,\ Q = Q_0 \end{cases}$$

$$I_0 = \frac{Q_0}{\sqrt{LC}}$$

Magnetic energy $= \frac{1}{2}LI^2 = \frac{1}{2}LI_0^2 \sin^2\omega t = \frac{1}{2}\frac{Q_0^2}{C}\sin^2\omega t$

5

Electric energy $= \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{1}{C}Q_0^2\cos^2\omega t$

5

These are equal when $\sin^2\omega t = \cos^2\omega t \Rightarrow \sin^2\omega t - \cos^2\omega t = 0$

$= \cos 2\omega t \Rightarrow t = \frac{\pi}{4\omega} \Rightarrow \boxed{t = \frac{1}{8}T}$

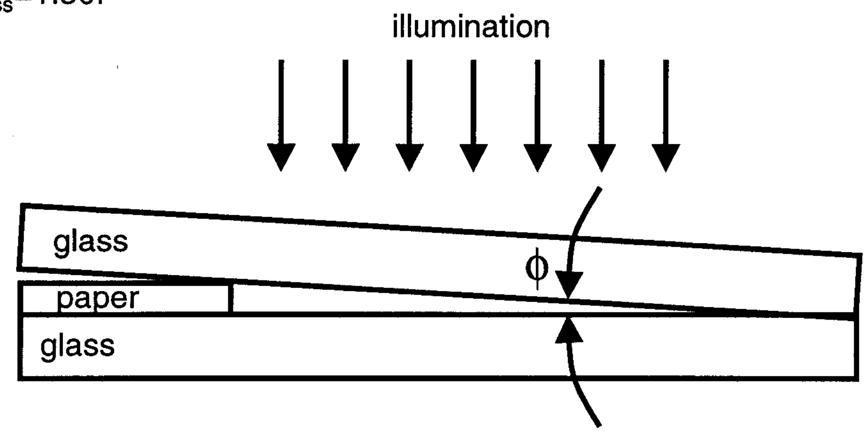
5

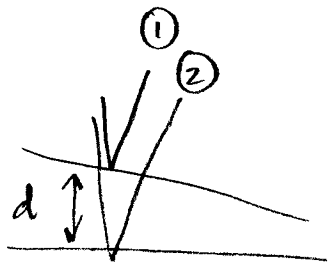
They are again equal at $t = \frac{3\pi}{4\omega} = \frac{3}{8}T$ or $\boxed{\frac{1}{4}T \text{ later}}$

[Equivalently $\sin^2\omega t = \frac{1}{2} \Rightarrow \sin\omega t = \frac{1}{\sqrt{2}} \Rightarrow \omega t = \frac{\pi}{4}$

$\Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4}\frac{T}{2\pi} = \frac{T}{8}$]

Problem 3 (20 points) Two plane glass slides are laid one on top of the other, with a thin piece of paper placed between them at one edge, so that a very small wedge of air is formed between them (see side view, below). The plates are illuminated at normal incidence with a monochromatic source of light with wavelength λ = 640 nm. You see 12 interference fringes per centimeter. Find the angle of the wedge ϕ. Note that n_{air} = 1.00, n_{glass}=1.50.





1 reflects with no phase shift (4)

2 reflects with π phase shift, and accumulates $2\pi \frac{d}{\lambda} \times 2$ from air space (4)

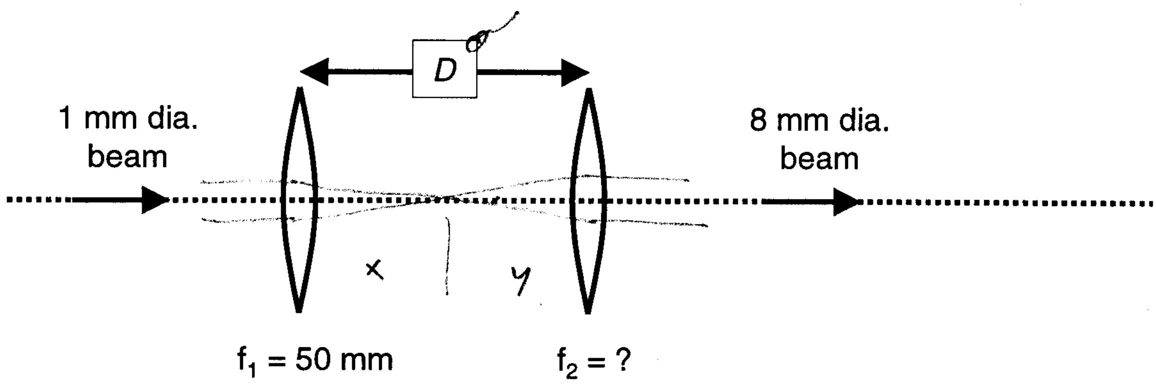
One fringe = 2π phase shift $\Rightarrow \frac{4\pi d}{\lambda} = 2\pi \Rightarrow d = \frac{\lambda}{2}$ (4)

$\frac{1 \text{ fringe}}{1/12 \text{ cm}} \Rightarrow \frac{d}{x} = \frac{\lambda/2}{1/12 \text{ cm}} = \tan\phi \approx \phi$ (4)

$$\boxed{\phi = \frac{6\lambda}{\text{cm}} = 3.8 \cdot 10^{-4} \text{ rad}}$$ (4)

$= 2.2 \cdot 10^{-2}$ deg

Problem 5 (20 points) A pair of converging lenses can be used to expand the diameter of a beam of light, for instance from a laser. A 1 mm diameter beam enters a converging lens with a focal length f_1= 50 mm, then passes through a second converging lens with a larger focal length f_2, emerging with a diameter of 8 mm. Find the focal length f_2 of the second lens, and the distance D of the second lens from the first.



1st lens brings light to focus at $s' = f_1 = 50\,mm$

Object for 2nd lens is at $s = D - 50\,mm$

Image from 2nd lens is at $s' = \infty$, so $s = f_2 = D - 50\,mm$

$$f_2 = D - 50\,mm$$

From drawing, $\frac{1\,mm}{x} = \frac{8\,mm}{y} \Rightarrow \frac{1\,mm}{f_1} = \frac{8\,mm}{f_2}$

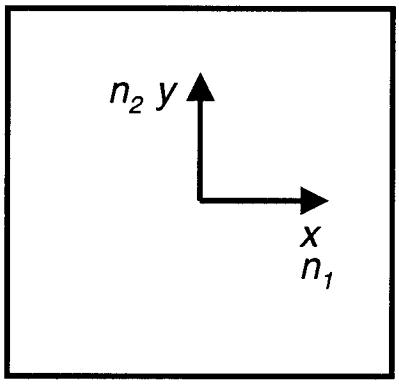
$$f_2 = 8 \cdot f_1 = 400\,mm$$

$$D = 450\,mm$$

Problem 6 (20 points) A **quarter wave plate** is an optical element that can transform circularly polarized light into linearly polarized light, and vice versa. One way to make a quarter wave plate is to use a slab of a *birefrigent* material, one which has a different index of refraction for the two linear polarizations of the electric field. For a slab with a thickness d into the page, the index of refraction for the electric field along x is n_1, while for that along y the index is n_2; assume $n_1 > n_2$. The slab will work as a quarter wave plate if the thickness d (into the page in the drawing) is such that light polarized along x emerges at the other side phase-shifted by $\pi/2$ (i.e. a quarter wavelength) compared to the light polarized along y.

(**a**) Find an expression for the smallest thickness d that will give this result, in terms of the indices of refraction and the vacuum wavelength of the light λ_0.

(**b**) For a plate made of material with $n_1 = 1.9$, $n_2 = 1.6$, and $\lambda_0 = 600$ nm, find d.



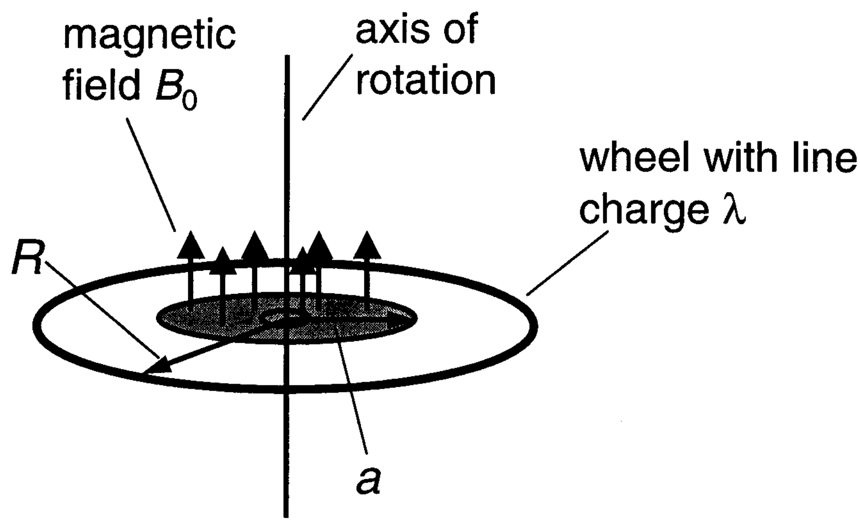
(a) $V_1 = \frac{c}{n_1}$ $V_2 = \frac{c}{n_2}$ $T = \frac{2\pi}{\omega}$ $\omega = \frac{2\pi c}{\lambda_0}$

$$\Delta\phi = \frac{2\pi}{T}(t_1 - t_2) = \omega d\left(\frac{n_1}{c} - \frac{n_2}{c}\right)$$

$$= \frac{2\pi d}{\lambda_0}(n_1 - n_2) \qquad \Delta\phi = \frac{\pi}{2} \rightarrow \boxed{d = \frac{\lambda_0}{4}\frac{1}{(n_1 - n_2)}}$$

(b) $d = \frac{600\text{nm}}{4}\frac{1}{(1.9-1.6)} = 500$ nm

Problem 6 (20 points) A line of charge with linear charge density λ (coulombs per meter) is glued to the rim of a wheel of radius R, which is suspended horizontally and free to rotate about its vertical axis. A uniform magnetic field B_0 pointing upwards is present in the central area, to a radius a. The field is turned off. What is the final angular momentum of the wheel? Note: It does not matter how fast the field is turned off!



$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt} = \pi a^2 \frac{dB}{dt}, \text{ uniform} \Rightarrow E = \frac{\pi a^2}{2\pi R}\frac{dB}{dt}$$

circumferential

$$\text{Torque} = \underbrace{2\pi R}_{\text{Circum.}} \underbrace{\frac{d\tau}{dl}}_{\text{torque per unit length}} = 2\pi R \cdot R \cdot \underbrace{\lambda E}_{\text{force per unit length}} = \lambda \pi a^2 R \frac{dB}{dt}$$

Final angular momentum $= \lambda \pi a^2 R B_0$