

*University of California, Santa Barbara*  
*Department of Physics*

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**PHYSICS 24**

**MIDTERM EXAM**

**WINTER 2004**

A. N. Cleland

10-10:50 am

Friday, February 6, 2004

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- **This exam is closed book, closed notes.**
  - **Calculators are allowed**
  - **Show all work clearly. Partial credit will be given if your thinking is clear.**
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Write your name:

Solutions

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Scoring:

Problem 1 \_\_\_\_\_ of 50

Problem 2 \_\_\_\_\_ of 50

Total \_\_\_\_\_ of 100

**Problem 1 (50 points).** A coaxial cable consists of an inner solid cylindrical conductor of radius  $a$ , surrounded by an empty annular space of radius  $b$ , which is in turn surrounded by an annular conducting cylinder of outer radius  $c$  (see drawing), all concentric with one another. Assume the conductors are infinite in length. A current  $I$  flows from left to right through the inner conductor, distributed uniformly through its cross section, and an equal current  $I$  flows out, from right to left, through the outer conductor (again with a uniform distribution). Find the magnetic field **magnitude** and **direction** for points a distance  $r$  from the center of the cable, with

- (a)  $r < a$  (inside the inner wire)
- (b)  $a < r < b$  (between the inner and outer conductors), and
- (c)  $c < r$  (outside the outer conductor).



Use Ampere's law: In all cases,  $B$  points tangent to a circle centered on the axis of the cable, clockwise rotation in the schematic.

$$(a) B = \mu_0 I r / 2\pi a^2$$

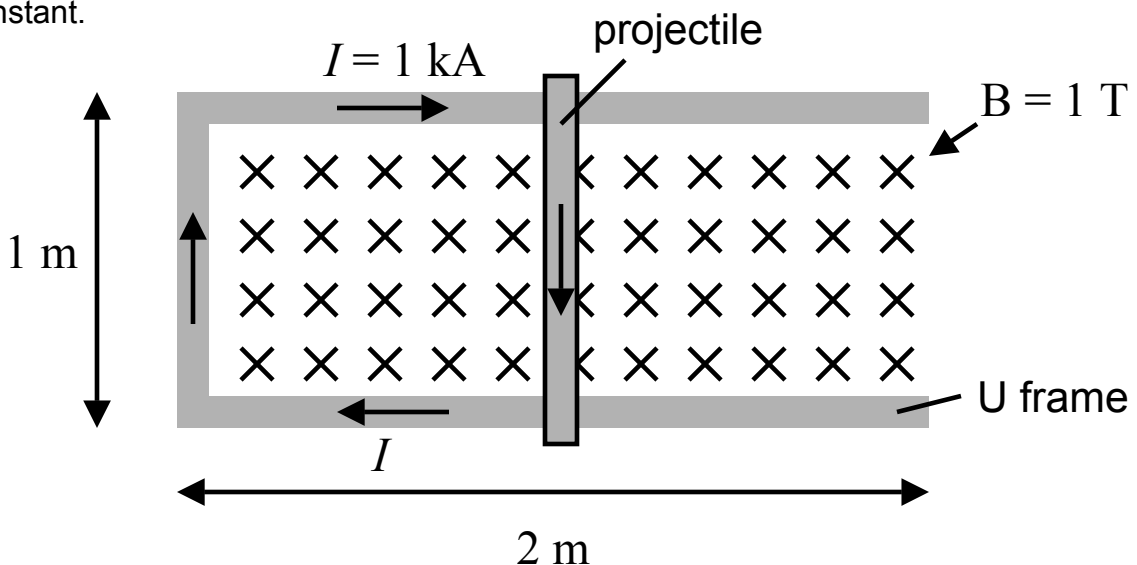
$$(b) B = \mu_0 I / 2\pi r$$

$$(c) B = 0$$

**Problem 2 (50 points)** A rail gun launches metal projectiles using magnetic fields. It consists of a frame in the shape of a horizontal  $U$ , along whose two parallel legs a bar of metal (the projectile) slides without friction. A source of current drives a current  $I$  of 1000 amperes around the loop formed by the  $U$  and the projectile, as shown below. The assembly is in a magnetic field of 1 Tesla, pointing into the page. The projectile has a length of 1 meter and a mass of 10 kg, and the  $U$  frame has a length of 2 meters. Ignore the magnetic field generated by the current, and assume the current source generates enough voltage to keep the current constant.

(a) Assuming the bar starts from rest at the midpoint of legs of the  $U$ , find its velocity (**direction** and **magnitude**) when it leaves the  $U$ .

(b) The source of current must supply a voltage even if the  $U$  frame and the bar have zero electrical resistance. Find the voltage as a function of time (up to the point where the bar leaves the frame) that the source of current must supply to keep the current constant.



(a) Force is  $F = I B L$ , so acceleration is  $I B L/m$ . To move 1 m ( $= L$ ) takes a time  $t_0 = (2L/a)^{1/2} = (2 m/I B)^{1/2}$ , so the velocity is  $v = a t_0 = (2 I B /m)^{1/2} L = (200)^{1/2} \text{ m/s} = 14 \text{ m/s}$

(b) The EMF is  $B dA/dt$  where  $A$  is the area  $= (1 \text{ m} + a t^2/2) 1 \text{ m}$ . Thus the EMF is  $B a t L = I B^2 L^2 t/m = (100 \text{ V/sec}) t$ . The voltage supplied must match this EMF.