Physics CS 36 **Spring 2014**

Set #4 - for Thurs May 1

<u>Read Ohanian</u> Ch. 5
<u>Read Feynman Vol. III</u> Ch. 1, Ch.2, Ch. 3

From Ohanian:

Ch. 5 Problems 17, 27, 28, 34.

Ch. 5 Problems 31 (find the standard deviation for Δx)

1. Show that if $\Delta x \Delta p = \frac{1}{2}\hbar$, the minimum energy of a simple harmonic oscillator is $\frac{1}{2}\hbar\omega$. What is the minimum energy in joules for a mass of 10^{-2} kg oscillating on a springof force constant 1.0 N/m?

2. A particle is on a table in a uniform gravitational field. The energy is $E = mgz + p^2/2m$, where $z = 0$ at the table. Classically, the minimum energy is $E = 0$. In quantum mechanics, because of the uncertainty principle, the particle cannot be at rest at a well-defined position. Assume then that the particle moves in a small range Δz above $z = 0$ and take the average height of the particle $\overline{z} = \frac{1}{2}\Delta z$ and apply the uncertainty principle in the form $\Delta z \Delta p \geq \hbar/2$.

a) Show that the average energy of the particle satisfies: $\overline{E} \geq \frac{1}{2}mg\Delta z + \frac{\hbar^2}{8m(\Delta z)^2}$.

b) Minimize $\overline{E}$ as a function of Δz and find $\overline{E}_{\text{min}}$ and $\overline{z}_{\text{min}}$ (the value of $\overline{z}$ that minimizes $\overline{E}$.)

3. A student drops a marble of mass m from a tower of height H. The student uses a very precise apparatus to aim the marble as well as possible so as to hit a particular crack in the sidewalk.

a) Show that the marble will, on average, miss the crack by a distance of order

$$d = \sqrt{\frac{h}{m}}\left(\frac{H}{g}\right)^{1/4}$$

where h is the Planck's constant and g is the acceleration due to gravity. (This formula ignores factors of 2 and π.)

b) Assuming reasonable values of H and m, estimate the values of d. How important are these quantum-mechanical effects?

4. From the uncertainty principle, estimate the minimum kinetic energy of an electron confined to a nucleus of size 5 fm (1 fm $= 10^{-15}$m). Note: This is a relativistic electron. Why?

5. Consider a continuous distribution of wave numbers $A(k)$ and consider the wavepacket at $t = 0$:

$$\psi(x) = \int_{-\infty}^{\infty} A(k)e^{ikx} dk$$

$$A(k) = \begin{cases} A_0 \ ; & |k - k_0| < \Delta k/2, \\ 0 \ ; & |k - k_0| > \Delta k/2 \end{cases}$$

a) Determine the wavefunction at $t = 0$: $\psi(x, 0)$.

b) Determine the probability density function $|\psi(x, 0)|^2$.

c) Sketch a graph of $|\psi(x, 0)|^2$ and discuss your result.

6. Consider a dispersive medium for which the relationship between the angular frequency $\omega = 2\pi\nu$ and the wave number $k = \frac{2\pi}{\lambda}$ is given by:

$$\omega(k) = \omega_0(k^3/6k_0^3 - k^2/2k_0^2 + 7k/16k_0)$$

where ω_0 and k_0 are positive constants having units of frequency and wavenumber respectively.

a) Sketch a graph of ω versus k.

b) Find the phase and group velocities as functions of k. Interpret your results.