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Mode competition in a semiconductor ring laser

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Abstract. In a homogeneously-broadened ring laser strong coupling between the counter-propagating travelling waves leads to stable oscillation in one direction at a time. Such behaviour has never been reported for semiconductor ring lasers in spite of the fact that their gain saturation has a homogeneous character. To explain this different nature of the mode competition, we have calculated the strength of the nonlinear coupling between the counter-propagating travelling waves in a semiconductor ring laser. Crucial elements of the calculation are the carrier diffusion in the amplifying medium and the large mismatch losses at the amplifier facets; the latter lead to a spatially non-uniform amplitude of the eigenmodes of the ring cavity. Additionally, we show how reflection from one of the facets leads to a linear coupling of the counter-propagating waves with a mixed conservative-dissipative nature. The final result of the mode competition depends on the interplay of linear and nonlinear coupling. Experimental results are presented for both a double-sided and a single-sided AR-coated semiconductor amplifier in a ring cavity. For the first ring the observed behaviour agreed well with theory, for the second ring only partial agreement was found.

1. Introduction

In this paper we investigate the competition between optical modes in a semiconductor ring laser. We restrict the discussion to a single longitudinal mode, which in an empty ring cavity is two-fold degenerate in both the direction of propagation (clockwise (c.w.)/counter clockwise (c.c.w.)) as well as the direction of polarization (transverse electric (TE)/transverse magnetic (TM)). This degeneracy can be lifted by the presence of reflecting elements or localized losses, which lead to a *linear* coupling of the counter-propagating travelling waves. This coupling is called linear, as it does not depend on the optical intensity. It determines whether the eigenmodes of the ring cavity are travelling waves, standing waves, or, as will be shown later, a combination of the two. Per definition linear coupling has no effect if one chooses the eigenmode basis to describe the dynamics of the ring cavity.

Additional coupling of the optical modes is provided by the intensity dependence of the optical gain, i.e. saturation of the amplifying medium. This coupling depends on the optical intensity and is therefore called *nonlinear* coupling. It determines the competition between the optical modes. Nonlinear coupling between two modes is called *strong* when the self saturation is smaller than the cross saturation, i.e. if the intensity in a certain mode leads to a smaller reduction of its own gain than of the gain in other modes. The nonlinear coupling is called *weak* when the self-saturation is larger than the cross saturation. If two linear eigenmodes are strongly coupled, only one of them will survive. In the case of weak coupling both modes can oscillate simultaneously [1].

If we consider the semiconductor gain medium as being homogeneously broadened, we would expect to find a strong nonlinear coupling between the counter-propagating travelling waves as, for example, in a ring dye laser. One would observe then a bistability in the direction of oscillation, i.e. the laser should oscillate either in a (c.w.) mode or in a (c.c.w.) mode. For a semiconductor ring laser this kind of bistability has never been reported so far [2–4]. Therefore, one has to face the fact that the nonlinear coupling of the modes in a semiconductor ring laser is more complicated than that in a simple homogeneously broadened medium.

A description of the semiconductor gain medium is complicated, mainly because the population is not confined to discrete levels, but is distributed over the valence and conduction *bands*. This complication would not be serious if the carrier distribution over these bands would always be in equilibrium, e.g. correspond to a Fermi–Dirac distribution at a certain temperature. The optical gain in the semiconductor medium would then be completely determined by the overall population inversion N. However, stimulated emission depopulates only those energy states of the bands for which the transition frequency fits the laser frequency. Intraband relaxation tries to restore the equilibrium carrier distribution over the bands, but the actual carrier distribution will depend on the optical intensity. Therefore the optical gain at the laser frequency generally depends on the population inversion N as well as the optical intensity I and can be written as $g(N, I)$. Note that for a normal three- or four-level laser the optical gain has *no* direct intensity dependence; it depends solely on the population inversion. Of course this population inversion is depleted as a result of stimulated emission, usually written as $N = N_0/[1 + (I/I_{sat})]$, where N_0 is the unsaturated population inversion and I_{sat} the saturation intensity, but this causes only an indirect intensity dependence of the gain. The direct dependence of the gain on the intensity is usually denoted as nonlinear gain [5, 6]. When discussing the nonlinear coupling of the modes we note that the nomenclature nonlinear gain can be somewhat confusing since in the case we will consider the nonlinear coupling involves only the indirect intensity dependence of the gain.

Intraband relaxation is an extremely fast process (typically 0·1 ps) and generally the deviations from the equilibrium carrier distribution are small. In contrast, interband relaxation, being the (radiative) decay of the overall population inversion, is much slower (typically 1 ns) and is the bottleneck in the decay towards the off-state equilibrium, with practically all electrons in the valence band. Therefore the direct intensity dependence of the gain, the nonlinear gain, is expected to be much smaller than the indirect intensity dependence, i.e. the gain depletion due to population changes induced by saturation. It will turn out in this paper that this is true for a large semiconductor ring laser, i.e. with a cavity length much larger than 1 m. The nonlinear coupling via the direct intensity dependence of $g(N, I)$ is then relatively unimportant; it is neglected in this paper as will be motivated in section 3. We consider only the nonlinear coupling via the N dependence of $g(N, I)$, where the optical intensity depletes the population inversion and thus indirectly reduces the optical gain. In this case a description in terms of the population inversion is valid. When effects of the direct intensity dependence of the gain have to be taken into account, a density matrix treatment is the preferred method.

The paper consists of a theoretical and an experimental part. In the theoretical part we develop a model for mode competition in a semiconductor ring laser consisting of an amplifying semiconductor chip and an external ring cavity. The basic ingredients of this theoretical section are as follows. We take into account the relatively strong mismatch losses associated with the coupling of the light into the waveguide of the semiconductor amplifier and the possible reflections at the amplifier facets. Due to the localized nature of the losses the amplitude of the eigenmodes in the amplifier is spatially non-uniform. This influences the mode competition which depends on the spatial overlap of the modes. We also consider the effects of diffusion on the spatial carrier distribution. This diffusion is shown to suppress all nonlinear coupling of the counter-propagating waves normally resulting from intensity induced gain or index gratings.

We will show that the vocabulary which has been developed to describe the mode competition in gas and dye lasers, e.g. spatial and spectral holeburning and population pulsations, becomes inadequate if one is to consider a semiconductor ring laser. Due to the large mismatch losses at the amplifier facets the eigenmodes of such a laser are neither travelling nor standing waves, but a mixture of the two, so that a terminology using index and gain gratings becomes misleading. The complexity arising from a proper treatment is discussed in this paper.

Observation of the time-dependence of the c.w. and c.c.w. output of a semiconductor ring laser is one way to study the nonlinear mode competition. Another diagnostic tool to study this mode competition is the production of a frequency splitting in the spectrum of the ring laser by means of backscattering. In the present work we have used this technique extensively; it has first been used for He–Ne-ring lasers [11] and ring dye lasers [12, 13]. Introducing a glass étalon into the ring cavity, aligned perpendicular to the mode axis, leads to a frequency splitting of the eigenmodes, which are standing waves in this case. However, the frequency splitting becomes only manifest in the spectrum of the corresponding ring laser if the nonlinear coupling between the standing-wave eigenmodes is weak as e.g. in a homogeneously-broadened ring laser, such as a ring dye laser; in that case both eigenmodes are found to oscillate simultaneously. If the competition would have been strong only one of the eigenmodes would have oscillated. Note that weak competition of the standing waves corresponds to strong competition of the travelling waves, and vice versa [11, 12]. For an inhomogenously-broadened ring laser the competition between the standing wave eigenmodes is generally strong and the étalon-induced frequency splitting is not observed.

We report experimental results on mode competition in two different semiconductor ring lasers. The first ring cavity was built around a double-sided anti-reflection (AR)-coated amplifier. This ring was studied all by itself and also with an additional backscattering element inserted into the ring. The second ring cavity was built around a single-sided AR-coated amplifier, where the facet reflection provided backscattering. Most (but not all) results are well explained by our theoretical model.

The paper is organized as follows. In section 2 we discuss theoretically the effects of large mode-mismatch losses at the facets and the effects of facet reflections. Section 3 treats the effect of spatial carrier diffusion and that of spatially non-uniform amplitudes of the eigenmodes on the strength of the nonlinear gain coupling. In section 4 the experimental results are presented. In section 5 the results are summarized and conclusions are given.

2. Ring resonator with backscattering, gain and losses

In this section the eigenmodes of a ring cavity with backscattering, gain and loss elements are calculated. The heart of the cavity (see figure 1) is formed by a semiconductor diode amplifier. Light emitted from each side of the amplifier waveguide is collimated by a lens and after reflection from four external mirrors focused by a second lens into the other side of the waveguide, thus establishing a semiconductor ring laser. We will restrict the discussion to one longitudinal mode of this ring laser. Such mode has, in principle, a twofold propagation degeneracy (c.w./c.c.w.) and a twofold polarization degeneracy (TE/TM). It is well established [14] that there is strong competition between the two polarization modes due to the large rate of elastic electron–electron collisions (typically $10^{13}\,s^{-1}$ [8]). This competition and the different waveguide losses of the semiconductor amplifier for the TE and TM polarizations will cause a stable oscillation in the lowest-loss mode (usually TE). Therefore we deal in this paper only with the twofold propagation degeneracy.

The reflectivity of the amplifier facets can be changed by dielectric coatings; for our devices it is either $\simeq 30\%$ (no coating) or $\simeq 0{\cdot}2\%$ (high quality AR-coating). Reflectivity of the amplifier facets leads to coupling of the counter-propagating travelling waves. In this paper we will restrict the discussion mainly to the case of zero reflectivities ($R_1 = R_2 = 0$) and the case where one of the facets is reflecting ($R_1 = 0, R_2 > 0$). If both facets are reflecting, the ring consists of two coupled cavities. An analytical treatment of such a system turns out to be very complex, as shown below, and is therefore omitted.

In the next two sub-sections we give alternative approaches for describing our ring cavity and calculating the eigenvalues and eigenmodes of the ring. The relation between the two formalisms is discussed in the third sub-section.

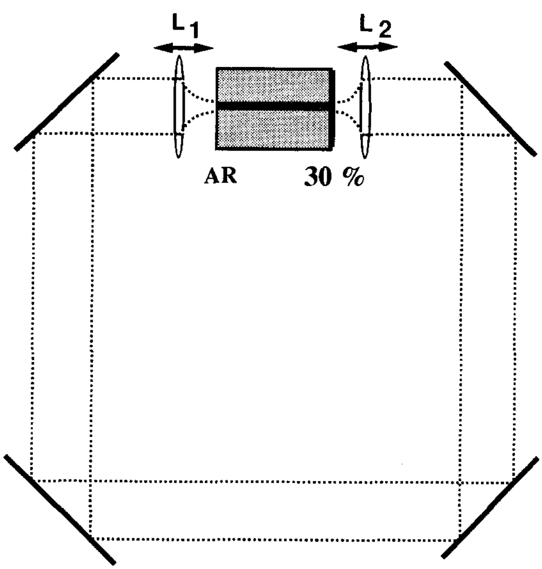

Figure 1. Experimental configuration of the semiconductor ring laser. The collimating lenses, denoted L_1 and L_2, are mounted on piezoelectric elements for fine tuning of the mismatch losses.

2.1. *Scattering matrix description*

In order to calculate the eigenmodes of the ring cavity we can use a scattering matrix description which relates, at a certain position along the ring, the incoming fields to the outgoing fields (see figure 2 (*a*)). Denoting the incoming complex field amplitudes by A and D and the outgoing complex field amplitudes by C and B, we see by comparing figure 2 (*a*) with figure 1 that D and B correspond to the c.c.w. and A and C to the c.w. travelling waves. The relation between these amplitudes can be written in terms of a scattering matrix **S**,

$$\begin{pmatrix} C \\ B \end{pmatrix} = \mathbf{S} \begin{pmatrix} A \\ D \end{pmatrix}. \tag{1}$$

Generally A, B, C and D are vectors since the electromagnetic field is a vector field. In our case, however, they are effectively scalar quantities since the polarization degeneracy of our ring is lifted as discussed above. The scattering matrix **S** can thus be represented by a 2×2 complex matrix.

We will now determine the form of the scattering matrix **S** for both the double sided AR-coated and a single sided AR-coated semiconductor amplifier in a ring cavity. Actually we consider only the situation where the amplifier has one reflecting facet (R) and one non-reflecting facet. By taking the limit $R \rightarrow 0$, this description also covers the case of two non-reflecting facets.

Two different losses arise when the light propagates outside the amplifier in the ring cavity from one facet to the other. First there are inevitable losses due to the limited numerical aperture and the non-perfect imaging of the lenses L_1 and L_2 (see

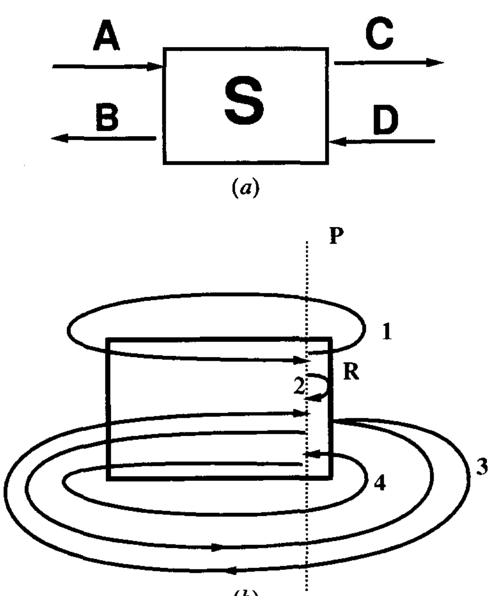

Figure 2. (*a*) The relation between the incoming and outgoing field amplitudes for an optical element described by the scattering matrix **S**. (*b*) The four different paths contributing to the scattering matrix of the semiconductor ring laser with a reflecting facet. P denotes the reference plane and R the partially reflecting surface.

figure 1) separately. These losses are denoted by an amplitude transmission coefficient κ where $\kappa<1$; they cannot be removed by proper alignment of the lenses. Second there are additional losses when the lenses are ot properly aligned with respect to each other; these losses are particularly sensitive to transverse misalignment. The lenses can be perfectly coupled in the sense that misalignment of one lens can be compensated by misalignment of the other. The reduction in roundtrip amplitude transmission due to imperfect compensation is denoted by f, where $f<1$.

In order to determine the various contributions to the outgoing field amplitudes, we choose a plane P, inside the semiconductor amplifier and infinitely close to the reflecting surface R, from where we start following the fields travelling around the ring cavity. We follow the fields around the cavity and stop when passing the starting point in P again. The different paths which can then be distinguished are shown in figure 2 (*b*).

For example, the outgoing c.w. amplitude C is made up by two contributions. The first contribution is due to path 1, as depicted in figure 2 (*b*), which corresponds to a roundtrip of the c.w. travelling field component A from plane P to P. This path is subject to some roundtrip and alignment losses. The second contribution is due to path 3, as depicted in figure 2 (*b*). This path corresponds to the c.c.w. travelling field component D that, starting from plane P, makes almost a complete roundtrip, is reflected by the facet R, travels back to the other facet and is coupled into the waveguide to reach P again. Via simple ray tracing it can be shown that for this path the incoupling losses at the facets are only due to the finite aperture of the lenses L_1 and L_2 and that they are not affected by a net transverse misalignment of these lenses. By determining likewise the contributions to the outgoing c.c.w. amplitude B (paths 2 and 4), we arrive at the following scattering matrix:

$$\mathbf{S}=\begin{pmatrix} tf\kappa^2 \exp(ikL+gl) & -r\kappa^2 \exp(2ikL+2gl) \\ r & tf\kappa^2 \exp(ikL+gl) \end{pmatrix}. \tag{2}$$

The amplitude reflectivity and transmission of the reflecting facet are denoted r and t, respectively. The length of the ring is denoted $L=L_b+nl$, where L_b is the effective optical length of the bulk cavity, and l and n are the length and refractive index of the semiconductor medium, respectively. The coefficient g is the optical gain coefficient of the semiconductor gain medium. The eigenmodes of the cavity can now be determined from the equation

$$\mathbf{S}\begin{pmatrix} A \\ D \end{pmatrix}=\begin{pmatrix} A \\ D \end{pmatrix}. \tag{3}$$

since the outgoing fields serve also as the incoming fields. Non-trivial solutions only exist if

$$\det(\mathbf{S}-\mathbf{I})=0, \tag{4}$$

where $\mathbf{I}$ is the unity matrix. Since $\mathbf{S}$ is a function of the cavity length, equation (4) determines the resonance frequencies of the ring cavity. From this equation it is found that the eigenmodes belonging to one longitudinal mode of the ring are separated in frequency by an amount

$$\Delta\nu=\frac{1}{\pi}\frac{c}{L}\arccos\left\{\frac{\kappa f t}{[(\kappa f t)^2+r^2]^{1/2}}\right\}, \tag{5}$$

where c/L is the free spectral range of the ring cavity. For each resonance frequency the corresponding eigenvector is determined by solving equation (3). The eigenvectors depend on the position along the ring, i.e. on the location of the plane P. For the scattering matrix given by equation (2), we find that the eigenvectors at the reflecting facet just inside the amplifier are given by

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ r \pm i\kappa f t \end{pmatrix}. \tag{6}$$

The laser threshold condition follows directly from equation (4) and is given by

$$\exp(-gl) = \kappa[(\kappa f t)^2 + r^2]^{1/2}. \tag{7}$$

In the limit of vanishing linear coupling and losses, i.e. $r=0$, $\kappa=f=1$, it follows from equations (5) and (6) that the eigenmodes become degenerate (no frequency splitting); they are standing waves with a spatial dependence given by a cosine and a sine function of the longitudinal coordinate z. In the limit of strong linear coupling, i.e. $r=1$, $t=0$, the precise values of κ and f are not vital. It follows from equation (6) that in this limit both eigenmodes have the same spatial dependence near the reflecting surface. In fact both eigenmodes have a node at the facet and a frequency separation of $c/2L$; the ring cavity has become a standing wave cavity.

Equation (5) predicts that the frequency splitting of the eigenmodes can be varied by changing the facet reflectivity r; this effect has been confirmed experimentally for the case of He–Ne and ring dye lasers [11, 12]. Moreover, equation (5) also predicts that the frequency splitting can be varied by changing the coupling losses (κ and f) while the facet reflectivity (r) remains constant. The minimum frequency splitting is predicted to occur for vanishing coupling losses, i.e. $\kappa f=1$. Thus both losses and reflection enter into the linear coupling which is responsible for the frequency splitting. Clearly semiconductor ring lasers are attractive to check the loss dependence of the frequency splitting since they can be made to operate with high losses (contrary to He–Ne and dye lasers).

If we would consider a ring laser with a semiconductor amplifier where *both* facets are reflecting, the situation is much more complex as already mentioned in the Introduction. Due to the presence of two reflecting facets, there is an infinite number of paths that has to be taken into account; light starting from P can in principle be reflected an infinite number of times by the facets before it leaks through and reaches P again. The corresponding **S** matrix for this case is therefore not easily obtained and the calculation of the cavity eigenmodes and eigenvalues is rather complex. In fact the ideal formalism to treat the case of several reflecting elements in the ring seems to be the transmission matrix model introduced by Lenstra and Geurten [15]. In this formalism the propagation of a field through a single element of the ring cavity is described by means of a transmission matrix **M**, defined by

$$\begin{pmatrix} C \\ D \end{pmatrix} = \mathbf{M} \begin{pmatrix} A \\ B \end{pmatrix}, \tag{8}$$

where A, B and C, D are the complex field amplitudes of the travelling waves to the left and the right of the element, respectively. Different optical elements in a ring can then be combined by simply multiplying the subsequent transmission matrices. Although this formalism looks elegant there is a serious pitfall: the different elements are described as *independent* of each other. It is impossible to use the transmission formalism when one is to describe the combined effect of mutually dependent elements, like the misalignment of the collimating lenses in the ring cavity of figure 1. Such elements have to be treated together, as we did in fact by directly writing down the **S** matrix of the complete ring (equation (2)).

2.2. *Dynamical matrix description*

The nature of the linear coupling of c.w. and c.c.w. modes in a ring cavity with backscattering is generally discussed on the basis of the dynamical evolution equations for the counter-propagating waves in the ring [11, 16]. In this subsection we will show how such a dynamical matrix description connects with the scattering matrix introduced above. The evolution of the field amplitudes at a given position in a cavity as a function of time is described by a dynamical matrix **K**, which is defined by

$$\dot{\mathbf{E}} \equiv \begin{pmatrix} \dot{E}_{cw} \\ \dot{E}_{ccw} \end{pmatrix} = -\mathrm{i}\mathbf{K}\cdot\mathbf{E}, \tag{9}$$

with $E_{ccw}=D$, $E_{cw}=A$ and where **K** is a 2×2 matrix containing gain, loss and coupling terms for the counterpropagating waves. For a passive ring, or a ring laser at or above threshold (i.e. no net gain or loss), the diagonal elements of the matrix vanish and the dynamical matrix has the general form

$$\mathbf{K} = \begin{pmatrix} 0 & W_1 \\ W_2 & 0 \end{pmatrix}. \tag{10}$$

This matrix describes the phase relation between the scattered travelling waves and thus determines directly the nature of the coupling. Of course the information about the nature of the coupling is the same as that contained in the scattering matrix **S** for the ring.

If the matrix **K** is Hermitian, i.e. $W_1 = W_2^*$, the energy is conserved in the scattering process and the coupling is called conservative. The eigenmodes are then standing waves with a frequency difference equal to $2|W_1|$. On the other hand, if **K** is anti-Hermitian, i.e. $W_1 = -W_2^*$, the coupling is dissipative. The standing wave eigenmodes then experience different losses and no frequency splitting occurs (i.e. frequency locking [11]). Generally we deal with a combination of conservative and dissipative coupling. In that case an analysis along the directions outlined above is still possible since the dynamical matrix **K** can always be written as the sum of a Hermitian and an anti-Hermitian one. Whether or not a frequency splitting occurs depends on the relative strength of the conservative to the dissipative coupling.

In analogy with the unitary evolution operator introduced in quantum mechanics, one can use the formal solution of equation (9) in order to derive the scattering matrix **S** which corresponds to a given dynamical matrix **K**. We must realise that the time t is *not* a continuous time parameter since the action of **K** does *not* represent a continuous change of the vector (E_{cw}, E_{ccw}) in time, but only the change after one complete round-trip time $T=L/c$. Therefore the formal solution to equation (9) ought to be written as:

$$\begin{pmatrix} E_{cw} \\ E_{ccw} \end{pmatrix}_{t=T} = \exp(-\mathrm{i}\mathbf{K}T) \begin{pmatrix} E_{cw} \\ E_{ccw} \end{pmatrix}_{t=0}. \tag{11}$$

By comparing with equation (3) we identify the scattering matrix as $\mathbf{S} \equiv \exp(-\mathrm{i}\mathbf{K}T)$; the general form of this scattering matrix is obtained by simply expanding the exponential $\exp(-\mathrm{i}\mathbf{K}T)$. As a result one finds

$$\mathbf{S} = \cos[T(W_1 W_2)^{1/2}] \times \begin{pmatrix} 1 & -\mathrm{i}\dfrac{W_1}{(W_1 W_2)^{1/2}} \sin[T(W_1 W_2)^{1/2}] \\ -\mathrm{i}\dfrac{W_2}{(W_1 W_2)^{1/2}} \sin[T(W_1 W_2)^{1/2}] & 1 \end{pmatrix}. \quad (12)$$

From this general scattering matrix for a lossless system, equation (12), one can see that for a conservative scattering process, as defined above, $\mathbf{S}$ becomes unitary, whereas for a dissipative scattering process $\mathbf{S}$ becomes pseudo-unitary [17].

When comparing, for a single-sided AR-coated semiconductor ring laser, the general scattering matrix given by equation (12) with that obtained in the previous sub-section (equation (2)), one finds the following expressions for the coupling coefficients W_1 and W_2,

$$W_1 = \pm \frac{c}{L} \frac{\exp(\mathrm{i}kL)}{[(\kappa f t)^2 + r^2]^{1/2}} \arctan\left(\frac{r}{\kappa f t}\right), \quad (13\,a)$$

$$W_2 = \pm \frac{c}{L} \frac{[(\kappa f t)^2 + r^2]^{1/2}}{\exp(\mathrm{i}kL)} \arctan\left(\frac{r}{\kappa f t}\right). \quad (13\,b)$$

The dynamical matrix of the single-sided AR-coated semiconductor ring laser is thus found to be

$$\mathbf{K} = \begin{pmatrix} 0 & W_1 \\ W_2 & 0 \end{pmatrix} = b \begin{pmatrix} 0 & z \\ z^{-1} & 0 \end{pmatrix}, \quad (14)$$

where $b = \pm(c/L)\arctan[r/(\kappa f t)]$ and $z = \exp(\mathrm{i}kL)/[(\kappa f t)^2 + r^2]^{1/2}$. It follows from equation (13 a, b) that in the limit of vanishing mismatch losses, i.e. $\kappa f \to 1$, the dynamical matrix becomes Hermitian, i.e. $W_1 = W_2^*$; the coupling due to the reflecting facet becomes therefore purely conservative. The coupling becomes non-conservative though if the reflectivity of the facet is unequal to zero *and* mismatch losses are present. We conclude that mismatch losses provide effectively a dissipative coupling of the c.w. and c.c.w. waves in the cavity. By separating the dynamical matrix with the values found for W_1 and W_2 using equation (13 a, b), into a Hermitian and an anti-Hermitian matrix one finds however that the mismatch losses also affect the conservative coupling. In fact one finds that the strength of the conservative coupling is always larger than that of the dissipative coupling, independent of the values of κ, f and r. Therefore one expects the linear eigenmodes to differ always in eigenfrequency.

3. Nonlinear gain and competition

3.1. *Gain saturation*

Whether the eigenmodes of the ring cavity are standing waves, travelling waves or a mixture of the two, is determined by the *linear* coupling. In this section we discuss how saturation of the amplifier gain leads to an additional *nonlinear* coupling of the (linear) eigenmodes of the ring cavity. The dynamics of the gain medium and the spatial form of the eigenmodes determine the strength of the nonlinear coupling.

As mentioned in the Introduction, in a semiconductor laser there are two varieties of saturation, related to two different relaxation processes. On a picosecond or subpicosecond timescale intraband scattering leads to a redistribution of the carriers over the conduction and valence band (intraband relaxation), while on a nanosecond timescale the overall population inversion decays through spontaneous and stimulated emission of radiation (interband relaxation). Most theoretical descriptions of the gain saturation associated with the fast intraband relaxation, are based on the density-matrix formalism [1, 7–10].

It is commonly assumed that the intraband relaxation process tries to establish a quasi-thermal equilibrium of the carriers over the conduction and valence band on a timescale of about 0·1 ps [7, 18]. The nature of this quasi-thermal equilibrium is still an open question. While some authors claim spectral hole burning to be of major importance [7, 10] others prefer a discussion in terms of dynamic carrier heating [18]. Actually, the carrier dynamics in AlGaAs might well be too complicated to be described by a single intraband relaxation rate.

The second relaxation process involved is the decay of the overall population inversion towards its off-state equilibrium via spontaneous and stimulated emission of radiation. Typical relaxation times of this (slow) interband relaxation process are about 1 ns. On this slow time scale the semiconductor medium is well characterized by a single quantity, the population inversion N, being the sum of the population differences of all connected states in the conduction and valence band.

Whether one has to choose a description of the nonlinear coupling in terms of the density matrix or the population inversion depends mainly on the physical size of the laser. For a solitary semiconductor laser, with a typical cavity length of 250 µm, the beat frequency between successive longitudinal modes is generally larger than 100 GHz. The population inversion is by far too slow to respond to this fast intensity beat. The carrier redistribution over the energy bands is orders of magnitude faster and does respond to the intensity beat. Therefore, the nonlinear mode coupling in solitary semiconductor lasers is usually described in terms of intraband relaxation and calculated via the density matrix [7–9].

The large semiconductor ring laser that we consider here (see section 2) has a free spectral range c/L of only 265 MHz. The frequency difference between the competing eigenmodes that we consider is a reasonably small fraction thereof (e.g. 20%, see equation (5)) so that the population inversion is expected to follow the resulting beat (e.g. 50 MHz). This allows a description in terms of the population inversion. An important advantage of such a description is that carrier diffusion can be incorporated easily. Similar considerations apply to the four-wave mixing process in semiconductor amplifiers. This process is also described in terms of the density matrix when the beat between the optical fields is fast ($\geqslant 100$ GHz) and in terms of the population inversion when the optical beat is slow [9, 19].

The calculation of the nonlinear coupling between the cavity eigenmodes is complicated by the effect of carrier diffusion. Since we consider only the population inversion the relevant timescale is that of the carrier lifetime. The diffusion length during the carrier lifetime is $(D\tau)^{1/2}$, where D is the carrier diffusion coefficient and τ is the carrier lifetime. In typical cases [21] $(D\tau)^{1/2} \approx 0{\cdot}7$ µm, which by far exceeds the period of the optical standing-wave pattern inside the medium, $\lambda/2n \approx 0{\cdot}13$ µm. The drastic effect thereof on the mode competition will be discussed in section 3.3.

In the rate equation approximation the time evolution of the population inversion $N(z, t)$ is given by [6, 20]

$$\frac{\mathrm{d}N}{\mathrm{d}t}+\frac{(N-N_0)}{\tau_s}+cIN=\lambda_{\text{pump}}+D\frac{\mathrm{d}^2N}{\mathrm{d}z^2}, \tag{15}$$

where τ_s is the spontaneous emission lifetime of the carriers, N_0 corresponds to the off-state (unpumped) equilibrium with basically all electrons in the valence band, c is a coupling constant and λ_{pump} is the pump rate that keeps the laser going. The second term at the right-hand side of equation (15) represents the carrier diffusion. It is treated in a one-dimensional fashion: we neglect diffusion in the x direction (normal to the active layer), because here the carriers are confined by the electronic band structure, and in the y direction (parallel to the active layer), because the width of the layer is much larger than the diffusion length. The second and third term on the left-hand side of equation (15) represent depopulation via spontaneous and stimulated emission, respectively. Far above threshold, depopulation via stimulated emission is an important decay channel and the population inversion $N(z, t)$ partially mimics the spatial and temporal dependence of the intensity $I(z, t)$. Notice that in the presence of two optical modes, labelled 1 and 2, the population inversion is driven by the individual mode intensities as well as by the beat between the two optical fields. The former perturbation is always static; the latter is dynamic if the mode frequencies are unequal and is then associated with the so-called population pulsations [1].

3.2. *Results in standard case*

The calculation of the mode competition from the perturbed population inversion now follows the lines of standard third-order theory [1, 20] if carrier diffusion is neglected (i.e. if $D=0$) and if the optical losses are relatively small. We first briefly review this procedure and subsequently point in the next sub-section at novel aspects which arise if carrier diffusion and/or optical losses are not negligible. Formal solution of equation (15) yields an expression for the population inversion in terms of the optical fields, which can be used to eliminate the population inversion from the field equations. This elimination introduces an intensity dependence in the optical gain, leading to nonlinear coupling between the optical modes. Following this procedure one finds that the self saturation of mode 1 is caused solely by population change induced by I_1 and that it is proportional to the overlap integral $\langle I_1 I_1\rangle$, where we have introduced the symbol

$$\langle\dots\rangle\equiv\frac{1}{l}\int_0^l \mathrm{d}z\dots, \tag{16}$$

to denote the integral taken over the amplifying medium. Similarly, the self saturation of mode 2 is proportional to the overlap integral $\langle I_2 I_2\rangle$. However, the cross-saturation of mode 1 by mode 2 (and vice versa) has two contributions. First of all I_2 induces population changes, which affect the evolution of I_1. This contribution is static and corresponds physically to spatial holeburning when the competition between standing waves is considered. Secondly the beat term $E_1E_2^*$ also induces changes in the population. These changes are dynamic when the eigenmode frequencies differ and are called population pulsations. Both these contributions to the cross saturation are generally of equal magnitude. The total cross saturation is then proportional to $2<I_1 I_2>$, where the factor 2 denotes the two equal contributions. Therefore, in the absence of diffusion the relative strength of the cross saturation θ, to the self saturation β, of mode 1 is found to be

$$\xi \equiv \frac{\theta}{\beta} = 2\frac{\langle I_1 I_2\rangle}{\langle I_1^2\rangle}\frac{\langle I_1\rangle}{\langle I_2\rangle}, \tag{17}$$

where the ratio $\langle I_1\rangle/\langle I_2\rangle$ is included for normalization; ξ is often called the competition parameter. Note that equation (17) is a special case of an expression for the competition parameter given in [10], which was derived for a solitary laser. For a ring laser with a homogeneously-broadened gain medium and low optical losses, one finds from equation (17) the standard result $\xi = 2$ for the nonlinear coupling between travelling-waves and $\xi = 2/3$ for that between standing waves.

3.3. *Effects of localized losses and carrier diffusion*

Equation (17), giving the ratio of cross-over self saturation is quite general and not limited to the condition of low optical losses. For our external-cavity semiconductor ring laser the optical losses are in fact rather large and well localized at the mirrors, lenses and amplifier facets. The losses are compensated by gain in the active medium. Therefore, the travelling-wave intensities are not constant, but grow exponentially inside the active medium and decline stepwise at the positions of the localized losses. This reduces the overlap between the intensity profiles of the travelling waves and thus reduces their nonlinear coupling (ξ). In other words, the nonlinear coupling between the original modes is changed since a standing-wave component is admixed into the travelling-wave modes. Equation (17) remains valid in the presence of large localized losses as long as diffusion can be neglected.

It should be noticed that a complication might arise, because in the semiconductor medium a change of the population inversion not only affects the gain, but also leads to a change in the refractive index. In principle both gain and refractive index variations, of a spatial and/or temporal nature, can scatter light from one mode into the other. In fact equation (17) is only as simple as it is in the limit that the population inversion can easily follow the frequency beat between the eigenmodes. In this limit, light scattered from the refractive index variations is out of phase with the incident optical field and therefore does not perturb its amplitude but only its phase, leading to mode-pulling. If the beat frequency would have been larger, the population inversion would have experienced a phase lag relative to the driving fields. The scattering efficiency from the dynamic gain variations would then be reduced, but the scattering from the related refractive index variations would no longer be out of phase, but would give an additional contribution to the nonlinear coupling [9]. We expect that for our semiconductor ring laser the frequency difference between the relevant eigenmodes is small enough to neglect the influence of the dynamic variations in the refractive index.

In semiconductor lasers carrier diffusion drastically changes the spatial dependence of the population inversion. Its effect can be most easily evaluated by expanding the terms that drive the population inversion (i.e. the static mode intensities I_1 and I_2 and the beat $E_1 E_2^*$), into their spatial Fourier components, which can be interpreted as spatial gratings. As a result of carrier diffusion a population inversion grating with wave-vector $\mathbf{k}$, $|\mathbf{k}| = k$ will be reduced by a factor $1/(1 + D\tau k^2)$ as compared to the situation without diffusion [7, 20]. Population gratings with a large wave-vector **k** will be washed out, while gratings with a small wave-vector are hardly affected. The effect of diffusion depends on the spatial profiles of the eigenmodes of the semiconductor ring laser and we will separately treat the cases of a double-sided and a single-sided AR-coated amplifier.

The eigenmodes of a ring laser containing an amplifier with two perfect AR-coatings are degenerate, since there is no coupling. We choose to calculate the mode competition in the travelling wave basis (c.w., c.c.w.). Each of the individual mode intensities has a spatially exponential profile inside the gain medium. This profile is burned into the population inversion by saturation; it is not affected by diffusion, since the length of the amplifier (typically 250 μm) is much larger than $(D\tau)^{1/2}$. However, the interference between the counter-propagating travelling waves also burns a spatial pattern into the inversion. This carrier density grating, with a period of half a wavelength ($\lambda/2n$), leads to a spatial modulation of the gain inside the amplifier, which scatters each of the travelling waves into the other, thus causing additional coupling between the travelling waves (remember the factor of 2 in equation (17)). Carrier diffusion will now wash out this 2**k** grating, thus reducing the cross-saturation coefficient θ without affecting the self-saturation coefficient β. Substituting exponential profiles for the travelling waves in an equation similar to equation (17), but now taking carrier diffusion into account, one finds that the coupling between the travelling waves is given by

$$\xi \mp \left[1+\frac{1}{1+D\tau(2k)^2}\right]\left(\frac{2gl}{\sinh 2gl}\right). \tag{18}$$

In the limit of slow carrier diffusion ($D\approx 0$) the original result (equation (17)) is recovered. Using $(D\tau)^{1/2}\approx 0{\cdot}7$ μm for AlGaAs [21] we find that carrier diffusion reduces the inversion grating to $\approx 10^{-3}$ of its maximum strength, leading to $\xi\approx 1{\cdot}001$ for a low-loss semiconductor ring laser ($g\approx 0$). In fact, ξ can become even less than unity if a practical value for the optical gain g is used in equation (18). This point is discussed further in the Experimental section 4.

The second example we treat is the semiconductor ring laser with a single-sided AR-coated amplifier. The optical field and intensity of the eigenmodes of the ring laser inside the amplifier can be found by combining equation (6), which gives the ratio of the left- and right-ward propagating fields E_+ and E_- at the reflecting facet, with the gain in the amplifying medium

$$\begin{aligned} E_\pm(z) &= \exp[g(z-l)]\exp[ik(z-l)] \\ &\quad + (r\pm i\kappa ft)\exp[-g(z-l)]\exp[ik(z-l)], \end{aligned} \tag{19 a}$$

$$\begin{aligned} I_\pm(z) &= \exp[2g(z-l)] + (r^2+\kappa^2 f^2 t^2)\exp[-2g(z-l)] \\ &\quad + 2[(r^2+\kappa^2 f^2 t^2)]^{1/2}\cos(2kz\mp\phi), \end{aligned} \tag{19 b}$$

$$\tan\phi = \kappa ft/r. \tag{19 c}$$

Notice that each of the eigenmode intensities is composed of two spatially exponential parts and a standing wave grating with uniform strength over the length of the amplifier. To evaluate the nonlinear coupling in the presence of carrier diffusion, it is essential to separately consider the population changes induced by I_1, I_2 and the beat $E_1E_2^*$ and to evaluate the effect of carrier diffusion on each of these terms. Because the eigenmodes now consist of a mixture of travelling and standing waves, the nonlinear coupling originates from both the smooth exponential change in the carrier density as well as from scattering of the $2\mathbf{k}$ grating. If the overlap integrals are calculated and the smoothing effect of diffusion on the $2\mathbf{k}$ grating is taken into account one finds for the competition parameter between the eigenmodes:

$$\xi = \frac{2\left[\dfrac{\sinh(2gl)}{2gl}\right]\{\exp(-2gl)+\exp(2gl)[r^2+(\kappa ft)^2]^2\}+4r^2\left(1+\dfrac{1}{1+4D\tau k^2}\right)}{\left[\dfrac{\sinh(2gl)}{2gl}\right]\{\exp(-2gl)+\exp(2gl)[r^2+(\kappa ft)^2]^2\}+2[r^2+(\kappa ft)^2]\left(1+\dfrac{1}{1+4D\tau k^2}\right)}. \quad (20)$$

Notice that the gain g is not an adjustable free parameter; it depends on the other four parameters (r, t, κ and f) via the laser threshold condition (equation (7)). We checked equation (20) by considering the limit of low reflection and low loss ($r \approx 0$, $\kappa ft \approx 1$, $g \approx 0$), where the eigenmodes are pure standing waves (see equation (17)). In this limit equation (20) reduces to $\xi = 2/3$ in the absence of diffusion; for an increasing amount of diffusion, ξ approaches 1 from below ($\xi \uparrow 1$).

4. Experiments

In this section we discuss experiments which we performed on various semiconductor ring lasers. The amplifiers that were used in our experiments were either double-sided or single-sided AR-coated. Thus we were able to investigate the nonlinear coupling of the travelling waves in a semiconductor ring laser, using a double-sided AR-coated amplifier, and the predicted frequency splitting of the eigenmodes, using an amplifier with only one AR-coating. The residual reflectivity of the AR-coatings was 0·2%.

First we discuss the experiments where a double-sided AR-coated semiconductor amplifier and bulk optics were used to form a ring cavity. The weakly index-guided amplifier chip (Philips VSIS AlGaAs laser, operating around 780 nm) was mounted in such a way that both facets were accessible [22]. The two AR-coated collimating lenses were placed on xyz-translation tables; final adjustments could be done using piezo-electric transducers. The length of the cavity was 1·17 m which corresponds to a free spectral range (FSR) of 265 MHz. The laser threshold was found to be 47 mA. The output of the laser was studied in the temporal domain, using a detector with a 4 GHz bandwidth and in the spectral domain using a 250 MHz Fabry–Perot interferometer with a finesse of $\simeq 300$. We observed that this ring laser operated single frequency in both directions simultaneously, with a laser linewidth of less than 800 kHz. Bistability between c.w. and c.c.w. travelling waves was never observed. These results are in agreement with previous reports on semiconductor ring lasers built in a similar way (see e.g. [2–4]).

The experimental observation that the laser always oscillated in both directions simultaneously can be theoretically explained by using equation (18) to calculate the mode competition between two counter-propagating travelling waves. Typical values for the losses in the two-sided AR-coated semiconductor ring laser lead to the estimates $\kappa = 0{\cdot}8$ and $f = 1$, making the single-pass gain $\exp(2gl) = \kappa^{-4} = 2{\cdot}44$. Combining these values with fast diffusion ($4D\tau k^2 \approx 10^3$) gives a calculated value for the competition parameter of less than one: $\xi \simeq 0{\cdot}88$. The combined effect of carrier diffusion and localized losses thus leads to weak coupling between the travelling wave eigenmodes of a semiconductor ring laser, as opposed to the strong coupling encountered in a ring dye laser ($\xi = 2$). This result explains why for semiconductor ring lasers bistability with respect to the direction of oscillation has never been observed experimentally.

Another mechanism that could possibly explain the absence of a bistable switching of the oscillation direction is that of unintentional linear dissipative coupling present somewhere in the cavity. Such dissipative coupling, e.g. due to localized losses somewhere in the cavity, can in principle suppress the nonlinear coupling [12, 13]. However, according to the theory discussed in section 2, a frequency splitting can still be obtained in this case when conservative coupling is introduced with a strength larger than that of the unintentional dissipative coupling. Therefore we applied strong additional linear conservative coupling, on top of that supplied by the residual reflectivity of the AR-coated amplifier facets, by inserting into the cavity a coated glass étalon, oriented perpendicular to the cavity mode axis. Several étalons were used, with intensity reflections of up to 50%. However, a frequency splitting was never observed. This result makes an unintentional dissipative killing of possibly strong competition of the counter-propagating travelling waves rather improbable.

In other experiments we looked more closely at the effects of intentional facet reflectivities. A single-sided AR-coated semiconductor laser was used in the same ring cavity as described above. The threshold injection current for this cavity was found to be 44 mA. The output of this semiconductor ring laser *did* show a frequency splitting for injection currents which were 3 mA or more above threshold; at lower currents the splitting was absent. The frequency splitting could be tuned continuously by changing the mismatch losses at the facets for a fixed injection current. If the injection current was raised further above threshold, typically about 8 mA, the ring laser started oscillating in several longitudinal modes of the ring cavity; the fundamental frequency splitting was still observed. These longitudinal modes of the ring cavity, however, were not grouped around one single longitudinal mode of the solitary amplifier cavity, but distributed over different modes of the latter. This indicates the presence of some residual coupling by the AR-coated facet, as apparently the amplifier modes are still rather well defined.

In the single-mode regime the frequency splitting of the oscillating mode was observed using a Fabry–Perot interferometer (FSR = 1·8 GHz, Finesse = 80). By changing the misalignment losses the splitting could be varied continuously from 52 to 114 MHz. Transverse misalignment proved a much more sensitive way to tune the frequency splitting than the longitudinal misalignment. Figure 3 shows a typical optical spectrum of the semiconductor ring laser, where the laser oscillated in a single longitudinal mode of the ring cavity. Besides the fundamental frequency splitting of the longitudinal mode we observed a higher-order sideband (see arrows), which is probably due to four-wave-mixing processes in the gain medium [9, 12, 19]. The frequency splitting was also observed as a beat by making a spectral analysis of the laser output intensity, using a 350 MHz detector and a RF-analyser. In figure 4 a typical RF-spectrum is shown; note that the vertical scale of the spectrum is logarithmic. In this spectrum the first higher-order harmonic of the frequency splitting also appears. Its intensity is about one tenth of that of the main beat.

The minimum value which was observed for the frequency splitting (i.e. 52 MHz) is in reasonable agreement with the theoretical prediction, i.e. $\Delta\nu_{min} = 60$ MHz, which follows directly from equation (5) with $\kappa = 0{\cdot}8$, $f = 1$, $r^2 = 0{\cdot}32$ and $L = 1{\cdot}17$ m. The theory also explains the experimental observation that the

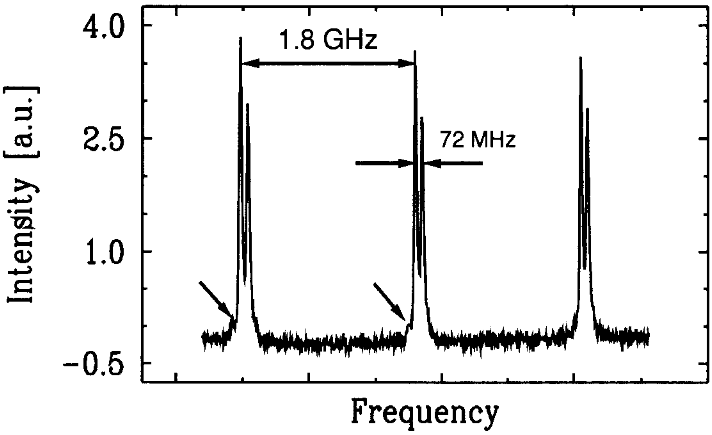

Figure 3. Optical spectrum of a single-sided AR-coated semiconductor ring laser, as detected with a Fabry–Perot interferometer with a free spectral range of 1·8 GHz. The frequency splitting is 72 MHz; also one higher-order harmonic of this splitting is visible (see arrows).

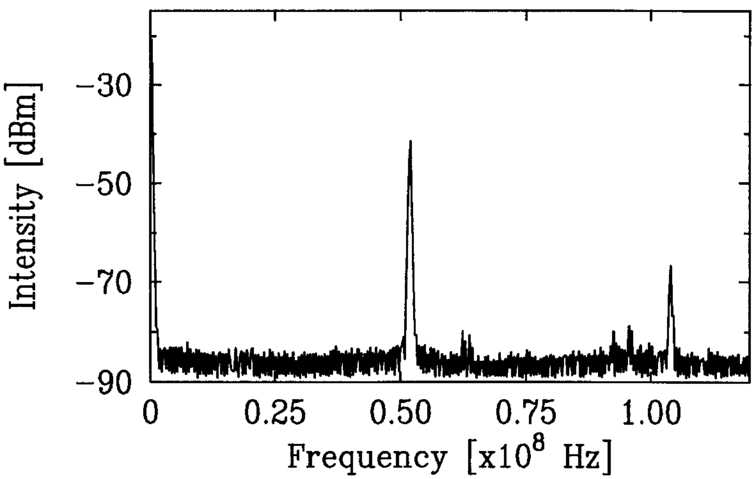

Figure 4. RF-beat spectrum of a single-sided AR-coated semiconductor ring laser as measured with a spectrum analyser and a detector with a bandwidth of 350 MHz. The fundamental frequency splitting is 52 MHz. A higher-order harmonic of this beat is clearly visible.

frequency splitting depends on the coupling losses, the splitting being smallest for the smallest losses. Experimentally the coupling losses are varied by changing the transverse positions of the piezo-mounted collimating lenses. The variation of the frequency splitting is shown in figure 5, where we plotted the frequency splitting as a function of the piezo voltage on either of the two collimating lenses. As expected from the theory in section 2, we found that the tunability of the beat shows roughly similar sensitivity to both lens misalignments. We also confirmed that a misalignment of one of the lenses could be compensated by repositioning the other lens.

The competition between the eigenmodes of a single-sided AR-coated semiconductor ring laser can be calculated using the expression for the competition

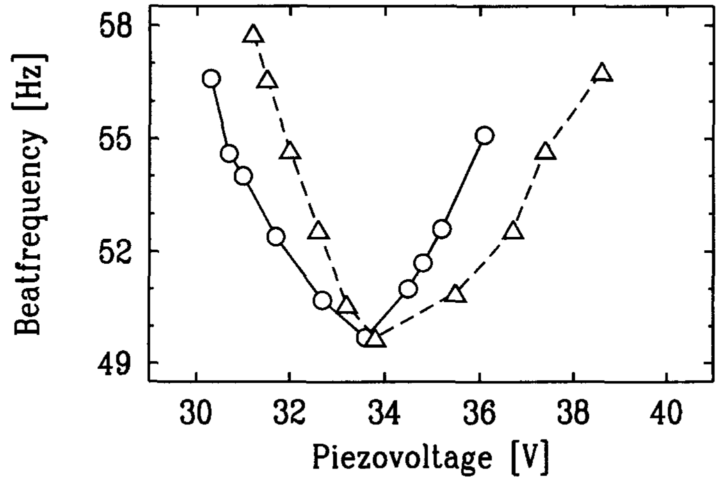

Figure 5. Frequency splitting as a function of the mismatch losses, which were changed by varying the voltage of the piezo elements used for transverse positioning of the collimating lenses. The circles refer to the position of L_1 and the triangles to that of L_2 (see figure 1).

parameter ξ as given by equation (20). Substituting typical values in equation (20), i.e. $\kappa = 0{\cdot}8, f = 1, r^2 = 0{\cdot}32$, together with a diffusion strength given by $4D\tau k^2 = 10^3$, we find $\xi = 1{\cdot}46$. This value for ξ predicts that the eigenmodes of the ring laser with a single-sided AR-coated amplifier should be strongly coupled. The laser is thus expected to oscillate in only one mode at a time, contrary to the experimental results, which show a frequency splitting.

However, one should realize that the general expression for the competition parameter equation (17) has been derived under the condition that the population pulsations induced by the beat of the eigenmodes is slow compared to the depletion time of the inversion, i.e. the inversion can follow the beat perfectly (see section 3). For the ring cavity used so far, this condition is only marginally fulfilled: the angular frequency splitting varies between $2\pi \times 52$ MHz and $2\pi \times 114$ MHz, while the interband carrier relaxation rate is about 1 GHz. Therefore the population pulsation will probably not be exactly in phase with the intensity oscillations. This might then explain why the nonlinear coupling between the eigenmodes of the ring is apparently weaker than that calculated above, using equation (20).

In order to verify the correctness of this assumption, we increased the length of our ring cavity in a subsequent experiment from 1·17 m to 6 m, thereby decreasing the beat frequency between the eigenmodes by a factor of five. This ring laser had a threshold current of 52 mA. The FSR of this cavity is 53 MHz and the minimum frequency splitting induced by the uncoated facet of the amplifier is expected to be about 11 MHz. In this experiment the ring laser oscillated most often in more than one longitudinal mode at a time. To suppress the multi-longitudinal mode oscillation we improved the spectral selection of the ring cavity using an additional coated glass étalon (FSR = 15 GHz, intensity reflectivity 20% per surface). This étalon had its normal slightly tilted with respect to the resonator beam axis. When pumped not too far above threshold the laser oscillated in a single longitudinal mode of the ring cavity in both directions; confirming our hypothesis, a frequency splitting was *not* observed in this case.

If the injection current was raised further above threshold, typically 8 mA, the laser started oscillating in several longitudinal modes, the number of which could be limited to only three at a time by tuning the étalon. By making again a spectral analysis of the laser output intensity, using the RF-analyser, we observed a frequency component at 66 MHz, which we interpret as the sum of the FSR of the long ring cavity (53 MHz) and the frequency splitting caused by the uncoated amplifier facet (assumed to be 13 MHz). This frequency could be varied continuously through transverse misalignment of the lenses, as was observed with the shorter ring cavity. In the RF-spectrum no component was observed at the frequency splitting itself (i.e. 13 MHz). These results can be explained if we assume for a moment that our model is still (partially) valid in the multi-mode case. This assumption means that the nonlinear coupling between the different longitudinal modes of the cavity is supposed to be weak compared to the nonlinear coupling between the (doublet) eigenmodes. A similar assumption was found to be valid in the case of a ring dye laser [12]. Apparently one of the eigenmodes within each doublet (the lower or higher frequency mode) is extinguished, consistent with the strong competition implied by the theoretical value $\xi = 1{\cdot}46$. This explains why the fundamental frequency splitting is not observed directly in the RF-spectrum but only when summed with the FSR of the ring cavity. The fact that we never observed a frequency component at the FSR of the cavity *minus* the frequency splitting (i.e. 40 MHz), is probably due to the nature of the nonlinear coupling between the different longitudinal modes.

5. Conclusions

We have studied the competition between the optical modes in a semiconductor ring laser both theoretically and experimentally. The theory predicts that the competition between the travelling waves in a double-sided AR-coated semiconductor ring laser is weak due to carrier diffusion and large mismatch losses. These losses lead to an exponential growth of the intensity profiles inside the semiconductor amplifier, thus affecting the mode overlap integrals which determine the strength of the mode competition. It is found that the competition between the counter-propagating travelling waves is weak, resulting in the ring laser oscillating in both directions simultaneously; this has been verified experimentally.

A linear theory has been developed in section 2 in order to calculate the effect of reflection of one of the facets on the mode spectrum of a semiconductor ring laser. This linear theory was substantiated by the experimental results. We found that a frequency splitting of the oscillating mode occurred, which could be varied by changing the coupling losses at the amplifier facets. The minimum observed frequency splitting was in good agreement with the theoretically calculated value. Finally, misalignment of one of the lenses could be compensated by repositioning the other lens, as predicted.

The theoretical model including the nonlinear gain predicts a strong competition between the eigenmodes for the case of a semiconductor ring laser with one reflecting amplifier facet, leading to extinction of one of the modes. At variance with this prediction, a frequency splitting was in fact observed in both the spectral and temporal domain for the first ring cavity we used in the experiments. We ascribe the discrepancy to the fact that for this ring cavity ($L = 1{\cdot}17$ m) the condition that the population pulsations should be slow compared to the carrier relaxation rate was only barely met. Violation of this condition is expected to result in a decreasing strength of the competition between the eigenmodes. This interpretation has been substantiated experimentally by using a larger ring cavity ($L = 6$ m) where the frequency splitting was indeed not observed.

It is interesting to note that from the point of view of mode competition, a semiconductor ring laser with cavity length L shows apparently different behaviour for $L \gg 1$ m as compared to $L \ll 1$ m. From a more general viewpoint, this difference can be seen as an example of the fact that an extended-cavity semiconductor laser can be categorized either as class A or a class B [23]; for a class A laser the evolution rate of the field is much smaller than that of the inversion whereas both rates are of the same order of magnitude for a class B laser.

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